Solving Equations Using Logarithms

Specification: AQA 7357: F4, F5 · Edexcel 9MA0: 6.4, 6.5

INDEPENDENT ## You do

A · $a^x = b$ and change of base

1 Solve, giving each answer to 3 s.f.:
(a) $2^x = 50$
(b) $6^x = 15$
(c) $0.8^x = 0.3$
(d) $10^x = 0.04$

(a) $x = \dfrac{\log 50}{\log 2} = 5.64$ — take logs; ÷ log 2

(b) $x = \dfrac{\log 15}{\log 6} = 1.51$ — take logs; ÷ log 6

(c) $x = \dfrac{\log 0.3}{\log 0.8} = 5.40$ — both logs negative

(d) $x = \log 0.04 = -1.40$ — base 10: x = log b

2 Solve, giving each answer to 3 s.f.:
(a) $3^{2x} = 70$
(b) $5^{x-4} = 12$
(c) $2^{3x+1} = 100$
(d) $7^{1-2x} = 30$

(a) $2x = \log_3 70 = 3.867\ldots \Rightarrow x = 1.93$ — the power is the log; ÷ 2

(b) $x - 4 = \log_5 12 \Rightarrow x = 4 + 1.544\ldots = 5.54$ — the power is the log; + 4

(c) $3x + 1 = \log_2 100 = 6.643\ldots \Rightarrow x = 1.88$ — − 1, then ÷ 3

(d) $1 - 2x = \log_7 30 = 1.747\ldots \Rightarrow x = -0.374$ — rearrange for x

3 Use the change of base formula to find, to 4 s.f.:
(a) $\log_3 17$
(b) $\log_5 0.7$
(c) $\log_{12} 1000$

(a) $\dfrac{\log 17}{\log 3} = 2.579$ — change of base

(b) $\dfrac{\log 0.7}{\log 5} = -0.2216$ — log 0.7 < 0

(c) $\dfrac{\log 1000}{\log 12} = \dfrac{3}{\log 12} = 2.780$ — log 1000 = 3

B · x in two powers

4 Solve, to 4 d.p.:
(a) $3^x = 2^{x+2}$
(b) $6^{x-1} = 5^x$

(a) $x \log 3 = (x + 2) \log 2$ — take logs of both sides
$x(\log 3 - \log 2) = 2 \log 2 \Rightarrow x = 3.4190$ — collect x terms; factorise

(b) $(x - 1) \log 6 = x \log 5$ — take logs of both sides
$x(\log 6 - \log 5) = \log 6 \Rightarrow x = 9.8275$ — collect x terms; factorise

5 Solve $5^{x+1} = 8^x$, giving an exact answer and the answer to 3 s.f.

$(x + 1) \log 5 = x \log 8 \Rightarrow x \log 5 + \log 5 = x \log 8$ — take logs; expand

$x = \dfrac{\log 5}{\log 8 - \log 5}$ — collect x terms; factorise

$= 3.42$ (3 s.f.) — calculator

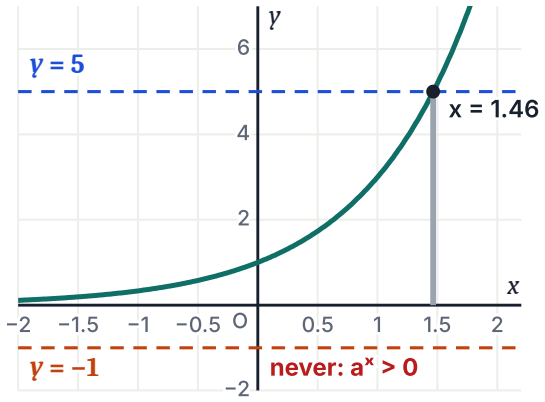
C · Quadratics in disguise Give exact answers, and decimals to 3 s.f.

6 Solve $2^{2x} - 9(2^x) + 8 = 0$.

Let $y = 2^x$: $y^2 - 9y + 8 = (y - 1)(y - 8) = 0$ — let y = 2ˣ; factorise
$2^x = 1 \Rightarrow x = 0$ — $2^0 = 1$
$2^x = 8 \Rightarrow x = 3$ — $2^3 = 8$

7 Solve $3^{2x} - 4(3^x) - 5 = 0$, giving your answer exactly and to 3 s.f.

Let $y = 3^x$: $y^2 - 4y - 5 = (y - 5)(y + 1) = 0$ — let y = 3ˣ; factorise
$3^x = 5 \Rightarrow x = \log_3 5 = 1.46$ (3 s.f.) — the power is the log
$3^x = -1$: no solution — reject: 3ˣ > 0



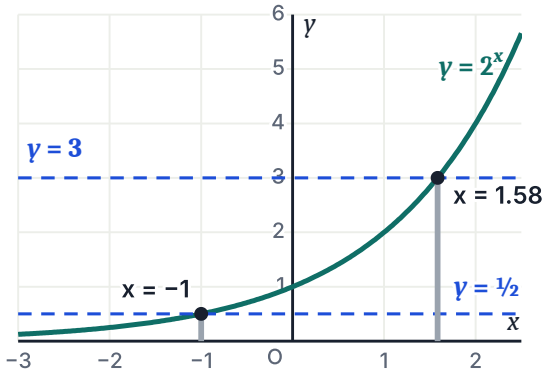
3ˣ = 5 at x = 1.46; 3ˣ = −1 is impossible

8 Solve $5^{2x} - 8(5^x) + 12 = 0$, giving your answers exactly and to 3 s.f.

Let $y = 5^x$: $y^2 - 8y + 12 = (y - 2)(y - 6) = 0$ — let y = 5ˣ; factorise
$5^x = 2 \Rightarrow x = \log_5 2 = 0.431$ (3 s.f.) — the power is the log
$5^x = 6 \Rightarrow x = \log_5 6 = 1.11$ (3 s.f.) — the power is the log

9 Solve $2^{2x+1} - 7(2^x) + 3 = 0$, giving your answers exactly and to 3 s.f.

$2y^2 - 7y + 3 = 0$ — $2^{2x+1} = 2(2^x)^2$
$(2y - 1)(y - 3) = 0$ — factorise
$2^x = \frac{1}{2} \Rightarrow x = -1$ — $2^{-1} = ½$
$2^x = 3 \Rightarrow x = \log_2 3 = 1.58$ (3 s.f.) — the power is the log

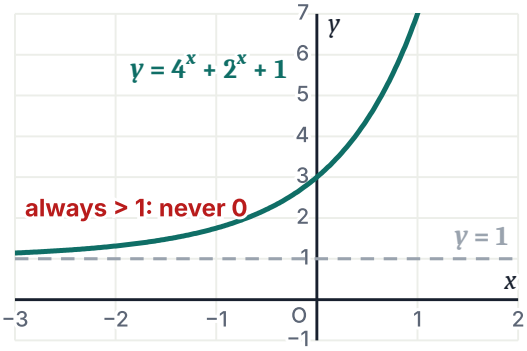


2ˣ = ½ at x = −1; 2ˣ = 3 at x = 1.58

D · Reasoning

10 Explain why $4^x + 2^x + 1 = 0$ has no real solutions.

Every term is positive: $4^x > 0$, $2^x > 0$. — aˣ > 0 always
So the left side is always more than 1: it cannot be 0. — sum > 1 > 0



always above y = 1, so never 0

11 Show that the solution of $3^x = 5$ can be written as $\frac{\ln 5}{\ln 3}$ and as $\frac{\log 5}{\log 3}$. Why do both give the same answer?

Take ln or log of both sides; the power law works in any base.

take logs: any base
Both equal $\log_3 5 = 1.465$ (4 s.f.) — change of base

12 Find the exact solution of $2^{3x-1} = 3$.

$3x - 1 = \log_2 3$ — the power is the log

$x = \dfrac{1 + \log_2 3}{3}$ — + 1, then ÷ 3

E · Exam-style question AQA-style. Use the mark scheme to self-assess.

13 (a) Show that the equation $9^x - 3^{x+2} + 14 = 0$ can be written as $y^2 - 9y + 14 = 0$, where $y = 3^x$. [2 marks]
(b) Hence solve $9^x - 3^{x+2} + 14 = 0$. Give your answers in exact form and to 3 s.f. [4 marks]

(a) $9^x = (3^x)^2 = y^2$ **B1** — $9 = 3^2$

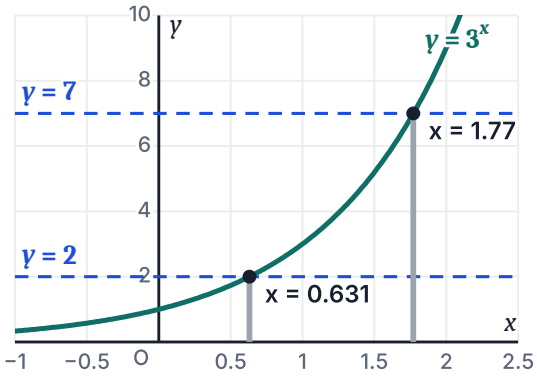
$3^{x+2} = 9 \times 3^x = 9y$ **B1** — split the power

(b) $(y - 2)(y - 7) = 0$ **M1** — factorise

$3^x = 2$ or $3^x = 7$ **A1** — $y = 3^x$

$x = \log_3 2 = 0.631$ (3 s.f.) **A1** — the power is the log

$x = \log_3 7 = 1.77$ (3 s.f.) **A1** — the power is the log



(b) $3^x = 2$ at x = 0.631; $3^x = 7$ at x = 1.77