Natural Logarithms

Specification: AQA 7357: F3, F4, F7 · Edexcel 9MA0: 6.3, 6.4, 6.7

INDEPENDENT

You do

A · Exact solutions

1 Solve:
(a) $e^x = 12$
(b) $e^{2x} = 7$
(c) $e^{x-3} = 4$
(d) $4e^{3x} = 20$

2 Solve:
(a) $\ln x = 5$
(b) $\ln(2x) = 3$
(c) $\ln(4x - 1) = 0$
(d) $3 \ln x + 2 = 8$

3 Solve, to 3 s.f.:
(a) $e^{0.5x} = 30$
(b) $\ln(x + 4) = 1.5$

B · Laws and quadratics

4 Solve $\ln x + \ln(x + 1) = \ln 12$.

5 Solve $2 \ln x = \ln(3x + 10)$.

6 Solve $e^{2x} - 9e^x + 20 = 0$.

7 Solve $e^{2x} + 2e^x - 15 = 0$.

8 Solve $2e^x - 3e^{-x} = 5$.

9 Solve $(\ln x)^2 - 5 \ln x + 4 = 0$.

C · Graphs and reasoning

10 Sketch $y = \ln(x - 2)$. State the asymptote and where it crosses the x-axis.

11 Sketch $y = 2 + \ln x$. Find the exact x-intercept.

12 Explain why $\ln x = -3$ has a solution but $e^x = -3$ does not.

D · Modelling with ln

13 A population is modelled by $P = 200e^{kt}$. When $t = 4$, $P = 500$. Find k exactly and to 3 s.f.

14 A substance decays according to $M = 50e^{-0.03t}$, where t is in years. Find its half-life, to 3 s.f.

E · Exam-style question AQA-style. Use the mark scheme to self-assess.

15 The concentration of a drug in the blood, C mg/l, t hours after a dose is modelled by $C = 8e^{-0.4t}$.
(a) State the initial concentration. [1 mark]
(b) Find the time when the concentration has fallen to 2 mg/l. Give an exact answer and an answer to 3 s.f. [3 marks]
(c) Show that $\frac{dC}{dt} = -0.4C$ and interpret the sign. [2 marks]