Logarithms and Non-linear Data

Specification: AQA 7357: F4, F6, F7 · Edexcel 9MA0: 6.4, 6.6, 6.7

INDEPENDENT **You do**

A · Log forms

1 For each model, write the log form. State what to plot, the gradient and the intercept.

(a) $y = 2x^5$

(b) $y = 0.5x^{-1}$

(c) $y = 3 \times 4^x$

(d) $y = 10 \times 0.5^x$

(a) $\log y$ vs $\log x$: gradient 5, intercept $\log 2 = 0.301$

power model: log–log

(b) $\log y$ vs $\log x$: gradient -1, intercept $\log 0.5 = -0.301$

power model: log–log

(c) $\log y$ vs x: gradient $\log 4 = 0.602$, intercept $\log 3 = 0.477$

exponential: log y vs x

(d) $\log y$ vs x: gradient $\log 0.5 = -0.301$, intercept 1

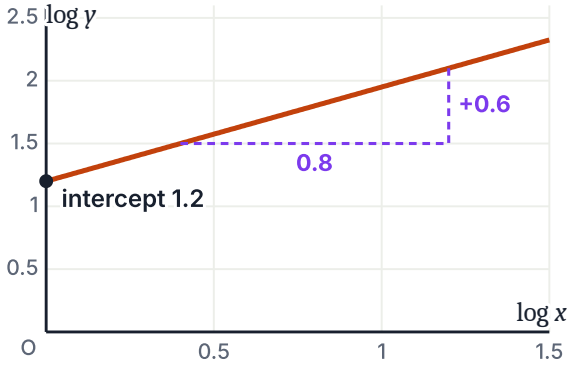
exponential: log y vs x

2 For $y = ax^n$, the graph of $\log y$ against $\log x$ has gradient 0.75 and intercept 1.2. Find a and n.

$\log y = n \log x + \log a$ — write the log form

$n = 0.75$ — gradient = n

$\log a = 1.2 \Rightarrow a = 10^{1.2} = 15.8$ — 10 to the power



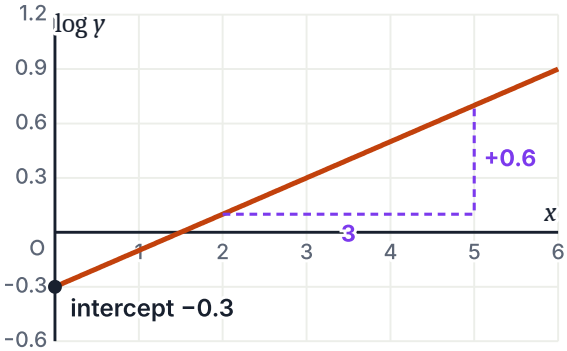
gradient n = 0.75

3 For $y = kb^x$, the graph of $\log y$ against x has intercept -0.3 and gradient 0.2. Find k and b.

$\log y = (\log b)\, x + \log k$ — write the log form

$\log k = -0.3 \Rightarrow k = 10^{-0.3} = 0.501$ — intercept = log k

$\log b = 0.2 \Rightarrow b = 10^{0.2} = 1.58$ — gradient = log b



intercept log k = −0.3

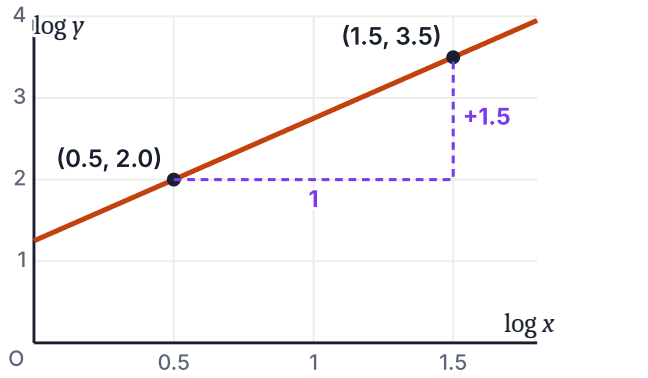
B · From two points

4 For $y = ax^n$, the line of $\log y$ against $\log x$ passes through $(0.5, 2.0)$ and $(1.5, 3.5)$. Find a and n.

$n = \dfrac{1.5}{1} = 1.5$ — gradient from two points

$c = 2.0 - 1.5(0.5) = 1.25$ — c = Y − mX

$a = 10^{1.25} = 17.8$ — 10 to the power

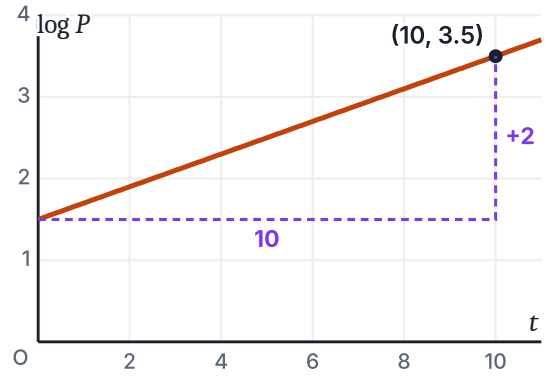


gradient 1.5 ÷ 1 = 1.5

5 For $P = kb^t$, the line of $\log P$ against t passes through $(0, 1.5)$ and $(10, 3.5)$. Find k and b.

$gradient = 0.2 \Rightarrow b = 10^{0.2} = 1.58$ — gradient = log b

$k = 10^{1.5} = 31.6$ — intercept = log k



gradient 2 ÷ 10 = 0.2

C · From data

6 The data follow $y = ax^n$.

x	1.5	3	5	8	12
y	4.6	22.5	72.9	215	546.2
$\log x$	0.18	0.48	0.70	0.90	1.08
$\log y$	0.66	1.35	1.86	2.33	2.74

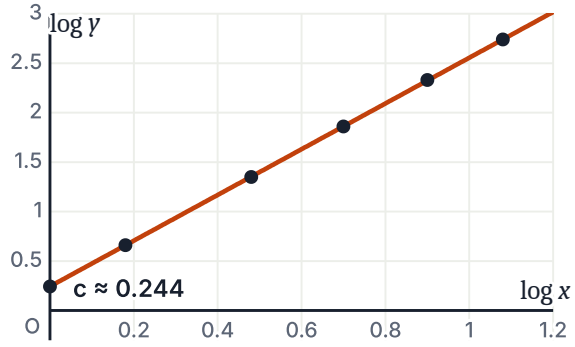
Use the first and last points to estimate a and n.

$n \approx \dfrac{2.74 - 0.66}{1.08 - 0.18} = 2.31$ — gradient: first and last points

$c \approx 0.66 - 2.31(0.18) = 0.244$ — substitute a point; keep 3 d.p.

$a \approx 10^{0.244} \approx 1.8$ — 10 to the power

$y \approx 1.8x^{2.3}$ — write the model



first and last points

7 The data follow $y = kb^x$.

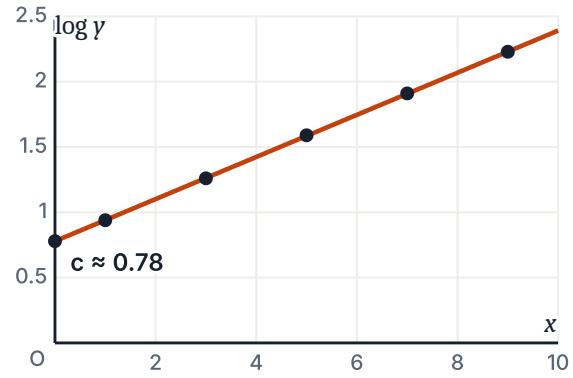
x	1	3	5	7	9
y	8.7	18.3	38.5	80.9	170
$\log y$	0.94	1.26	1.59	1.91	2.23

Use the first and last points to estimate k and b.

$gradient \approx \dfrac{2.23 - 0.94}{9 - 1} = 0.161$ — gradient: first and last points

$b \approx 10^{0.161} \approx 1.45$ — 10 to the power

$c \approx 0.94 - 0.161 = 0.779 \Rightarrow k \approx 6.0$ — substitute a point; 10 to the power



first and last points

8 It is thought that $y = ax^n$.

x	2	3	5	8	12	20
y	12.6	22.3	54.1	108	217	478
$\log x$	0.30	0.48	0.70	0.90	1.08	1.30
$\log y$	1.10	1.35	1.73	2.03	2.34	2.68

(a) Complete the table (2 d.p.).
(b) Plot $\log y$ against $\log x$ and draw a line of best fit.
(c) Use your line to estimate a and n.

(a) Reveal the $\log x$ and $\log y$ rows. — take logs of the data

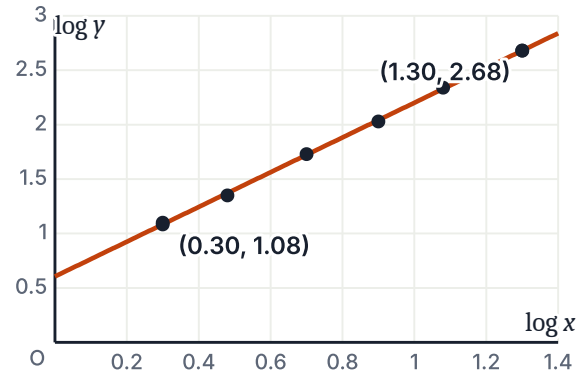
(b) The points lie close to a straight line. — plot; line of best fit

(c) The line passes through about $(0.30, 1.08)$ and $(1.30, 2.68)$ — two points far apart on the line

$n \approx \dfrac{2.68 - 1.08}{1.30 - 0.30} \approx 1.60$ — gradient = n

$c \approx 1.08 - 1.60(0.30) \approx 0.60 \Rightarrow a \approx 10^{0.60} \approx 4.0$ — intercept = log a

$y \approx 4.0x^{1.6}$ — your line may give slightly different values



line of best fit

9 The number of cells, N, in a culture t hours after it is started is thought to follow $N = kb^t$.

t	0	1	2	3	4	5	6
N	51	61	80	96	123	151	195
$\log N$	1.708	1.785	1.903	1.982	2.090	2.179	2.290

(a) Complete the table (3 d.p.).
(b) Plot $\log N$ against t and draw a line of best fit.
(c) Use your line to estimate k and b.

(a) Reveal the $\log N$ row. — take logs of the data

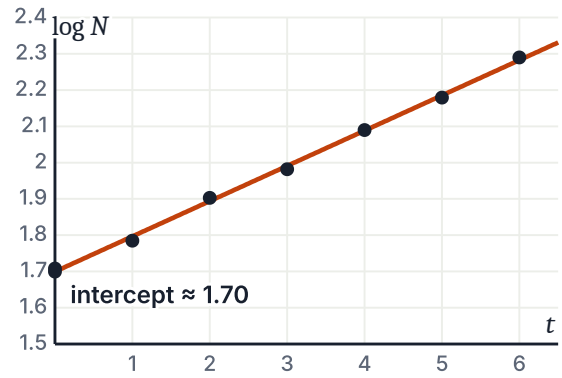
(b) The points lie close to a straight line. — plot; line of best fit

(c) Intercept ≈ 1.70, gradient ≈ $\frac{2.28 - 1.70}{6 - 0} \approx 0.097$ — read the line

$k \approx 10^{1.70} \approx 50$ — intercept = log k

$b \approx 10^{0.097} \approx 1.25$ — gradient = log b

$N \approx 50 \times 1.25^t$ — your line may give slightly different values



line of best fit

D · Reasoning

10 Explain why $\log y$ against x is a straight line for $y = kb^x$, but $\log y$ against $\log x$ is not.

$\log y = \log k + x \log b$: $\log y$ is a linear function of x.

multiplication law; power law

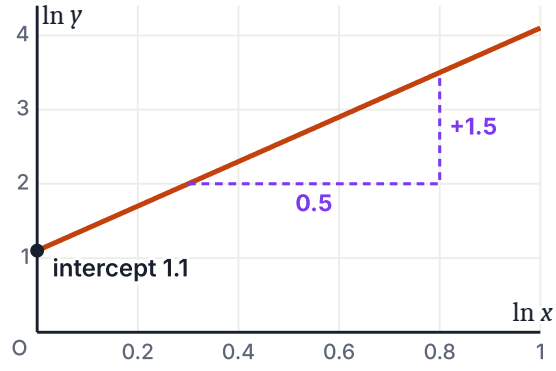
In terms of $X = \log x$, $x = 10^X$, so $\log y = \log k + 10^X \log b$: not linear.

$x = 10^X$: not linear

11 For $y = ax^n$, the graph of $\ln y$ against $\ln x$ has gradient 3 and intercept 1.1. Find a and n.

$\ln y = n \ln x + \ln a$ — the log form with ln

$n = 3, a = e^{1.1} = 3.00$ — ln: use e to the power



gradient n = 3; intercept ln a

12 For the data below, decide whether $y = ax^n$ or $y = kb^x$ is the better model. Find its constants.

x	1	2	3	4	5
y	6	12	24	48	96
$\log x$	0.00	0.30	0.48	0.60	0.70
$\log y$	0.778	1.079	1.380	1.681	1.982

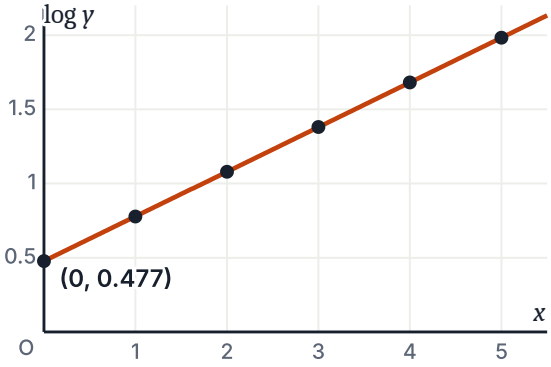
Each time x goes up by 1, $\log y$ goes up by 0.301: equal steps. — compare the differences

So $\log y$ against x is a straight line: $y = kb^x$. — straight against x: exponential

$\log b = 0.301 \Rightarrow b = 2$ — gradient = log b

$\log y = 0.301x + c$: $0.778 = 0.301 + c \Rightarrow c = 0.477$ — substitute (1, 0.778)

$k = 10^{0.477} = 3$, so $y = 3 \times 2^x$ — 10 to the power



straight against x

E · Exam-style question AQA-style. Use the mark scheme to self-assess.

13 The number of bacteria, N, t hours after a culture is started is recorded.

t	1	2	3	4	5
N	60	90	135	203	304
$\log N$	1.778	1.954	2.130	2.307	2.483

It is thought that $N = ab^t$.

(a) Show that this gives a linear relationship between $\log N$ and t. [2 marks]

(b) Complete the $\log N$ row (3 d.p.). [1 mark]

(c) Using the first and last points, estimate a and b. [3 marks]

(d) Use the model to predict N after 8 hours, and comment on its reliability. [2 marks]

(a) $\log N = \log a + \log b^t$ **M1** — multiplication law

$\log N = (\log b)\,t + \log a$: $Y = mX + c$ **A1** — power law: Y = mX + c

(b) e.g. $\log 60 = 1.778$; the row as revealed **B1** — take logs: 3 d.p.

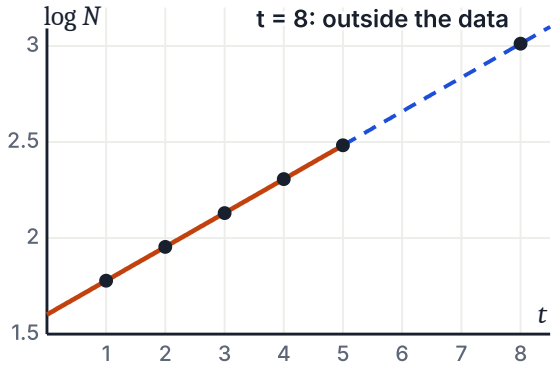
(c) $\text{m} = \dfrac{2.483 - 1.778}{4} = 0.176$ **M1** — gradient: first and last points

$b = 10^{0.176} = 1.50$ **A1** — 10 to the power

$c = 1.778 - 0.176 = 1.602 \Rightarrow a = 40.0$ **A1** — substitute a point

(d) $N = 40 \times 1.5^8 \approx 1030$ **B1** — substitute t = 8

Extrapolation beyond the data: less reliable (growth may slow). **E1** — extrapolation



(d) extrapolation