Logarithms and Non-linear Data

Specification: AQA 7357: F4, F6, F7 · Edexcel 9MA0: 6.4, 6.6, 6.7

INDEPENDENT

You do

A · Log forms

1 For each model, write the log form. State what to plot, the gradient and the intercept.
(a) $y = 2x^5$
(b) $y = 0.5x^{-1}$
(c) $y = 3 \times 4^x$
(d) $y = 10 \times 0.5^x$

2 For $y = ax^n$, the graph of $\log y$ against $\log x$ has gradient 0.75 and intercept 1.2. Find a and n.

3 For $y = kb^x$, the graph of $\log y$ against x has intercept -0.3 and gradient 0.2. Find k and b.

B · From two points

4 For $y = ax^n$, the line of $\log y$ against $\log x$ passes through $(0.5, 2.0)$ and $(1.5, 3.5)$. Find a and n.

5 For $P = kb^t$, the line of $\log P$ against t passes through $(0, 1.5)$ and $(10, 3.5)$. Find k and b.

C · From data

6 The data follow $y = ax^n$.

x	1.5	3	5	8	12
y	4.6	22.5	72.9	215	546.2
$\log x$					
$\log y$					

Use the first and last points to estimate a and n.

7 The data follow $y = kb^x$.

x	1	3	5	7	9
y	8.7	18.3	38.5	80.9	170
$\log y$					

Use the first and last points to estimate k and b.

8 It is thought that $y = ax^n$.

x	2	3	5	8	12	20
y	12.6	22.3	54.1	108	217	478
$\log x$						
$\log y$						

(a) Complete the table (2 d.p.).
(b) Plot $\log y$ against $\log x$ and draw a line of best fit.
(c) Use your line to estimate a and n.

9 The number of cells, N, in a culture t hours after it is started is thought to follow $N = kb^t$.

t	0	1	2	3	4	5	6
N	51	61	80	96	123	151	195
$\log N$							

(a) Complete the table (3 d.p.).
(b) Plot $\log N$ against t and draw a line of best fit.
(c) Use your line to estimate k and b.

D · Reasoning

10 Explain why $\log y$ against x is a straight line for $y = kb^x$, but $\log y$ against $\log x$ is not.

11 For $y = ax^n$, the graph of $\ln y$ against $\ln x$ has gradient 3 and intercept 1.1. Find a and n.

12 For the data below, decide whether $y = ax^n$ or $y = kb^x$ is the better model. Find its constants.

x	1	2	3	4	5
y	6	12	24	48	96
$\log x$					
$\log y$					

E · Exam-style question

AQA-style. Use the mark scheme to self-assess.

13 The number of bacteria, N, t hours after a culture is started is recorded.

t	1	2	3	4	5
N	60	90	135	203	304
$\log N$					

It is thought that $N = ab^t$.

(a) Show that this gives a linear relationship between $\log N$ and t. [2 marks]

(b) Complete the $\log N$ row (3 d.p.). [1 mark]

(c) Using the first and last points, estimate a and b. [3 marks]

(d) Use the model to predict N after 8 hours, and comment on its reliability. [2 marks]