Locating Roots

Specification: AQA 7357: I1 · Edexcel 9MA0: 9.1

HOMEWORK

Consolidation

1 Show that $x^3 - 3x^2 - 2 = 0$ has a root between $x = 3.1$ and $x = 3.2$.

AQA I1 · Edexcel 9.1

2 Show that $x - 3\cos x = 0$ (x in radians) has a root between $x = 1.1$ and $x = 1.2$.

AQA I1 · Edexcel 9.1

3 The equation $x^3 + 3x - 5 = 0$ has one root, α. Show that $\alpha = 1.154$ correct to 3 decimal places.

AQA I1 · Edexcel 9.1

4 $f(x) = \dfrac{x-1}{x-2}$, $x \neq 2$. Without a calculator:

(a) find $f(1.5)$ and $f(3)$

(b) explain why $f(x) = 0$ has no root in $1.5 < x < 3$.

AQA I1 · Edexcel 9.1

5 $f(x) = 10x^2 - 10x + 2.4$. Without a calculator:

(a) show that $f(0)$ and $f(1)$ are both positive

(b) find $f(0.5)$ and explain what it shows

(c) solve $f(x) = 0$ to check.

AQA I1 · Edexcel 9.1

6 (a) On the same axes, sketch $y = 2^x$ and $y = x + 3$.

(b) State the number of roots of $2^x - x - 3 = 0$.

(c) Show that the negative root lies between -2.9 and -2.8.

AQA I1 · Edexcel 9.1 (graph sketching)

7 Show that the curve $y = x^2 + 4\cos x$ (x in radians) has a stationary point with x-coordinate between 1.8 and 1.9.

AQA I1, G3 · Edexcel 9.1, 7.3

8 $f(x) = x^3 - 2x^2 - 1$.

(a) Complete the table.

(b) Write down an interval of width 0.1 that contains the root of $f(x) = 0$.

x	2	2.1	2.2	2.3	2.4
$f(x)$					

AQA I1 · Edexcel 9.1

9 The equation $e^{-x} - x = 0$ has a root α. Show that $\alpha = 0.567$ correct to 3 decimal places.

AQA I1 · Edexcel 9.1

10 $f(x) = x^3 - 3x + 2$.

(a) Show that $f(0.9)$ and $f(1.1)$ are both positive.

(b) Show that $f(x) = (x-1)^2(x+2)$.

(c) Explain why no interval around $x = 1$ shows a change of sign.

AQA I1 · Edexcel 9.1

11 *AQA-style.* $f(x) = \ln 2x + x^2 - 3$, $x > 0$.

(a) Show that $f(x) = 0$ has a root α between 1.4 and 1.5. [2 marks]

(b) Show that $\alpha = 1.40$ correct to 2 decimal places. [3 marks]

(c) By considering $f'(x)$, explain why $f(x) = 0$ has exactly one root. [2 marks]

AQA I1, G3 · Edexcel 9.1, 7.3 · AQA-style

12 *Edexcel-style.* $f(x) = e^x - 3x - 1$.

(a) Show that $f(x) = 0$ has a root α in the interval $[1.9, 2.0]$. [2 marks]

(b) Find the exact x-coordinate of the stationary point of $y = f(x)$ and show that its y-coordinate is negative. [3 marks]

(c) A student says: "$f(-1) > 0$ and $f(2) > 0$, so $f(x) = 0$ has no roots between -1 and 2." Use the stationary point to explain why the student is wrong, and write down a root of $f(x) = 0$ other than α. [2 marks]

AQA I1, G3 · Edexcel 9.1, 7.3 · Edexcel-style