Locating Roots

Specification: AQA 7357: I1 · Edexcel 9MA0: 9.1

INDEPENDENT **You do**

A · Showing a root

Show that $f(x) = 0$ has a root in the given interval.

1 $f(x) = x^3 + 3x - 7$, $1 < x < 2$

2 $f(x) = e^x - x - 4$, $1.7 < x < 1.8$

3 $f(x) = \ln x + 2x - 5$, $2.1 < x < 2.2$

4 $f(x) = \sqrt{x} - \cos x$ (x in radians), $0.6 < x < 0.7$

5 $f(x) = xe^x - 2$, $0.8 < x < 0.9$

6 $f(x) = x^3 + x - 5$.
(a) Complete the table.
(b) Between which two consecutive values of x is the root of $f(x) = 0$?

x	1.3	1.4	1.5	1.6	1.7
$f(x)$					

B · To a given accuracy

Write down the bounds, substitute them, conclude.

7 Write down the two values of x you would substitute to show that:
(a) $\alpha = 2.47$ to 2 d.p.
(b) $\alpha = 0.836$ to 3 d.p.
(c) $\alpha = -1.3$ to 1 d.p.
(d) $\alpha = 1.070$ to 3 d.p.

8 The equation $x^3 + x - 5 = 0$ has a root α (question 6). Show that $\alpha = 1.516$ correct to 3 d.p.

9 The equation $2\cos x - x = 0$ (x in radians) has a root α. Show that $\alpha = 1.030$ correct to 3 d.p.

10 The equation $x^4 - 2x - 5 = 0$ has a root α between 1.7 and 1.8. Show that $\alpha = 1.70$ correct to 2 d.p.

C · Reasoning

Explain fully. Q11–Q13: no calculator.

11 $f(x) = \dfrac{3}{x-1} + 2$, $x \neq 1$.
(a) Show that $f(0)$ and $f(2)$ have opposite signs.
(b) Solve $f(x) = 0$.
(c) Explain why $f(x) = 0$ has no root in $0 < x < 2$.

12 $f(x) = 4x^2 - 12x + 9$.
(a) Find $f(1)$ and $f(2)$.
(b) Factorise $f(x)$.
(c) Explain why a change of sign can never locate the root of $f(x) = 0$.

13 $f(x) = x^3 - 3x + 1$.
(a) Show that $f(0)$ and $f(2)$ have the same sign.
(b) By finding $f(1)$, show that $f(x) = 0$ has at least two roots in $0 < x < 2$.

14 $f(x) = \tan x - x$ (x in radians).
(a) Show that $f(1.5)$ and $f(1.7)$ have opposite signs.
(b) Explain why this does not show that $f(x) = 0$ has a root in $1.5 < x < 1.7$.

D · Show that

15 (a) On the same axes, sketch $y = e^x$ and $y = 6 - x$.
(b) Explain why $e^x + x - 6 = 0$ has exactly one root, α.
(c) Show that $1.5 < \alpha < 1.6$.
(d) Show that $\alpha = 1.503$ correct to 3 d.p.

16 (a) Show that the curve $y = e^x - x^3$ has a stationary point with x-coordinate β between 0.9 and 1.
(b) Show that $\beta = 0.91$ correct to 2 d.p.

17 (a) On the same axes, sketch $y = e^x$ and $y = 3 - x^2$.
(b) State the number of roots of $e^x + x^2 - 3 = 0$.
(c) Show that the negative root lies between -1.7 and -1.6.

E · Exam-style question

AQA-style. Use the mark scheme to self-assess.

18 $f(x) = 2x^3 - 5x^2 + 1$.
(a) Show that $f(x) = 0$ has a root α in the interval $[2.4, 2.5]$. [2 marks]
(b) Show that $\alpha = 2.414$ correct to 3 decimal places. [3 marks]
(c) Show that $(2x - 1)$ is a factor of $f(x)$. Hence find the exact value of α. [3 marks]