Iteration

Specification: AQA 7357: I2 · Edexcel 9MA0: 9.2

HOMEWORK Consolidation

1 Use $x_{n+1} = \sqrt[3]{4 - x_n}$ with $x_0 = 1$ to find x_1, x_2 and x_3.

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2 (a) Use $x_{n+1} = e^{-x_n}$ with $x_0 = 0.5$ to find x_1, x_2 and x_3.

(b) The iterates converge to α. Show that $\alpha e^{\alpha} = 1$.

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3 $f(x) = x^3 - 8x + 3$.

(a) Show that $f(x) = 0$ can be written as $x = \sqrt{8 - \frac{3}{x}}$, $x > 0$.

(b) Use $x_{n+1} = \sqrt{8 - \frac{3}{x_n}}$ with $x_0 = 3$ to find x_1, x_2, x_3.

(c) Show that $f(-3) = 0$. Hence find the exact root that the iterates approach.

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4 (a) Use $x_{n+1} = \sqrt{3x_n + 2}$ with $x_0 = 3$ to complete the table (4 d.p.).

(b) The iterates converge to α. Show that $\alpha^2 - 3\alpha - 2 = 0$ and find the exact value of α.

n	0	1	2	3	4
x_n	3				

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5 The equation $x^2 - 3x + 2 = 0$ is rearranged as $x = \frac{x^2 + 2}{3}$.

(a) With $x_0 = 1.5$, find x_1, x_2, x_3.

(b) With $x_0 = 2.5$, find x_1, x_2, x_3.

(c) Solve $x^2 - 3x + 2 = 0$ and describe what each sequence does.

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6 (a) Explain why $x_{n+1} = \ln(4 - x_n)$ cannot be used with $x_0 = 4$.

(b) Use it with $x_0 = 1$ to find x_1, x_2, x_3.

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7 The curve $y = e^x - 4x^2$ has a stationary point with x-coordinate between 3 and 4.

(a) Show that this x-coordinate satisfies $x = \ln 8x$.

(b) Use $x_{n+1} = \ln 8x_n$ with $x_0 = 3.5$ to find x_1, x_2, x_3.

AQA I2 · Edexcel 9.2 (stationary points)

8 $f(x) = x^3 - x^2 - 5$.

(a) Show that $f(x) = 0$ has a root α between 2.1 and 2.2.

(b) Show that $f(x) = 0$ can be written as $x = \sqrt[3]{x^2 + 5}$.

(c) Use $x_{n+1} = \sqrt[3]{x_n^2 + 5}$ with $x_0 = 2$ to find x_1, x_2, x_3.

(d) Show that $\alpha = 2.116$ correct to 3 d.p.

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9 No calculator. $x_{n+1} = \frac{6}{x_n + 1}$, $x_0 = 1$.

(a) Find x_1, x_2 and x_3.

(b) The iterates converge to α. Find α.

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10 $e^x - 5x = 0$ has roots 0.2592 and 2.5426.

(a) Use $x_{n+1} = \frac{e^{x_n}}{5}$ with $x_0 = 1$ to find x_1, x_2, x_3.

(b) Use $x_{n+1} = \ln 5x_n$ with $x_0 = 2$ to find x_1, x_2, x_3.

(c) Which root does each iteration approach?

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11 *AQA-style.* $f(x) = x^3 - 4x^2 + 2$.

(a) Show that $f(x) = 0$ has a root α in the interval $[3.8, 3.9]$. [2 marks]

(b) Show that $f(x) = 0$ can be rearranged into the form $x = 4 - \frac{2}{x^2}$. [2 marks]

(c) Use $x_{n+1} = 4 - \frac{2}{x_n^{\,2}}$ with $x_1 = 4$ to find x_2 and x_3, giving your answers to three decimal places. [2 marks]

(d) Explain why $x_1 = 0$ cannot be used. [1 mark]

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12 *Edexcel-style.* $f(x) = \ln(2x + 1) - x + 1$, $x > -\frac{1}{2}$. The equation $f(x) = 0$ has a root α with $\alpha > 2$.

(a) Show that $f(x) = 0$ can be written as $x = 1 + \ln(2x + 1)$. [1 mark]

(b) Use $x_{n+1} = 1 + \ln(2x_n + 1)$ with $x_0 = 3$ to find x_1, x_2 and x_3 to 4 decimal places. [3 marks]

(c) By choosing a suitable interval, show that $\alpha = 2.924$ correct to 3 decimal places. [3 marks]

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