# Iteration

Specification: AQA 7357: I2 · Edexcel 9MA0: 9.2

**INDEPENDENT** **You do**

**A · Iterating** Find $x_1$, $x_2$ and $x_3$ to 4 decimal places.

**1** $x_{n+1} = \sqrt[3]{x_n + 4}$, $x_0 = 1$

**2** $x_{n+1} = 3 - \ln x_n$, $x_0 = 2$

**3** $x_{n+1} = 1 + \frac{1}{2}\cos x_n$ ($x_n$ in radians), $x_0 = 1$

**4** $x_{n+1} = \ln(x_n + 5)$, $x_0 = 2$

**5** $x_{n+1} = \frac{x_n^{\,2} + 3}{5}$, $x_0 = 1$

**6** (a) Use $x_{n+1} = \sqrt[3]{10 - 2x_n}$ with $x_0 = 2$ to complete the table (4 d.p.).
(b) Describe how the values approach the root.

| $n$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $x_n$ | 2 | | | | |

**B · Rearranging** Show that each equation can be written in the form given.

**7** $x^3 - 7x + 3 = 0$ as $x = \frac{x^3 + 3}{7}$

**8** $e^x + 3x - 6 = 0$ as $x = \ln(6 - 3x)$, $x < 2$

**9** $x^3 + 2x^2 - 9 = 0$ as $x = \frac{3}{\sqrt{x + 2}}$, $x > 0$

**10** $x^3 - 2x^2 - 4 = 0$ as:
(a) $x = 2 + \frac{4}{x^2}$
(b) $x = \frac{2}{\sqrt{x - 2}}$, $x > 2$

**C · Reasoning** Explain fully. Q11 and Q12: no calculator.

**11** $x_{n+1} = \frac{1}{2}(x_n + \frac{5}{x_n})$, $x_0 = 2$.
(a) Find $x_1$ as a fraction.
(b) The iterates converge to $\alpha$. Show that $\alpha^2 = 5$ and write down the exact value of $\alpha$.

**12** $x_{n+1} = 4 - \frac{3}{x_n}$, $x_0 = 2$.
(a) Find $x_1$ and $x_2$.
(b) The iterates converge to $\alpha$. Show that $\alpha^2 - 4\alpha + 3 = 0$.
(c) Solve this equation. Which solution is $\alpha$?

**13** $x_{n+1} = \sqrt{7 - 4x_n}$.
(a) Explain why $x_0 = 2$ cannot be used.
(b) With $x_0 = 1$, find $x_1$, $x_2$, $x_3$.
(c) What happens when you try to find $x_4$?

**14** $x^3 + x - 4 = 0$ has one root $\alpha$, $1.3 < \alpha < 1.4$.
(a) Show that it can be written as $x = \frac{4}{x^2 + 1}$.
(b) Use $x_{n+1} = \frac{4}{x_n^{\,2} + 1}$ with $x_0 = 1.4$ to find $x_1$ to $x_4$.
(c) Does the iteration converge to $\alpha$? Explain.

**D · Show that**

**15** $f(x) = x^3 - 3x^2 - 5$ has one root, $\alpha$.

(a) Show that $f(x) = 0$ can be written as $x = 3 + \dfrac{5}{x^2}$.

(b) Use $x_{n+1} = 3 + \dfrac{5}{x_n^{\,2}}$ with $x_0 = 3.4$ to find $x_1$, $x_2$, $x_3$.

(c) Show that $\alpha = 3.426$ correct to 3 d.p.

**16** $f(x) = \ln x + 2x - 5$ has a root $\alpha$ between 2.1 and 2.2.

(a) Show that $f(x) = 0$ can be written as $x = \dfrac{5 - \ln x}{2}$.

(b) Use $x_{n+1} = \dfrac{5 - \ln x_n}{2}$ with $x_0 = 2$ to find $x_1$, $x_2$, $x_3$.

(c) Show that $\alpha = 2.12$ correct to 2 d.p.

**17** The curve $y = x^2 + 4\cos x$ ($x$ in radians) has a stationary point at $x = \beta$, $1.8 < \beta < 1.9$.

(a) Show that $\beta$ satisfies $x = 2\sin x$.

(b) Use $x_{n+1} = 2\sin x_n$ with $x_0 = 2$ to find $x_1$, $x_2$, $x_3$.

(c) Show that $\beta = 1.90$ correct to 2 d.p.

**E · Exam-style question** AQA-style (AQA starts from $x_1$). Use the mark scheme to self-assess.

**18** $f(x) = x^2 - 3 - \ln(x + 1)$, $x > -1$.

(a) Show that $f(x) = 0$ has a root $\alpha$ between 2 and 2.1. [2 marks]

(b) Show that $f(x) = 0$ can be rearranged into the form $x = \sqrt{3 + \ln(x + 1)}$. [2 marks]

(c) Use $x_{n+1} = \sqrt{3 + \ln(x_n + 1)}$ with $x_1 = 1.5$ to find $x_2$ and $x_3$, giving your answers to three decimal places. [3 marks]

(d) Explain why $x_1 = -1$ cannot be used. [1 mark]