Staircase and Cobweb Diagrams

Specification: AQA 7357: I2 · Edexcel 9MA0: 9.2

HOMEWORK Consolidation

1 (a) Use $x_{n+1} = \sqrt{x_n + 7}$ with $x_0 = 2$ to find x_1, x_2, x_3. Would the diagram be a staircase or a cobweb?
(b) The iterates converge to α. Show that $\alpha^2 - \alpha - 7 = 0$ and find α exactly.

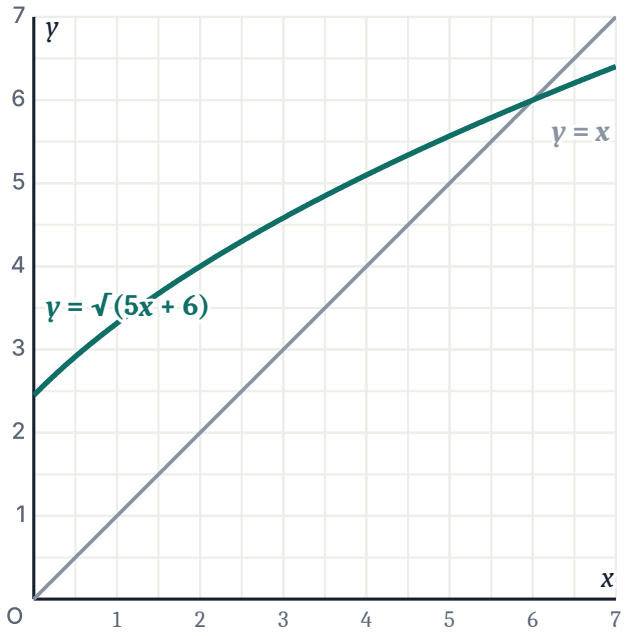
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2 Use $x_{n+1} = \dfrac{9}{x_n + 2}$ with $x_0 = 1$ to find x_1, x_2, x_3. Would the diagram be a staircase or a cobweb? Does it converge?

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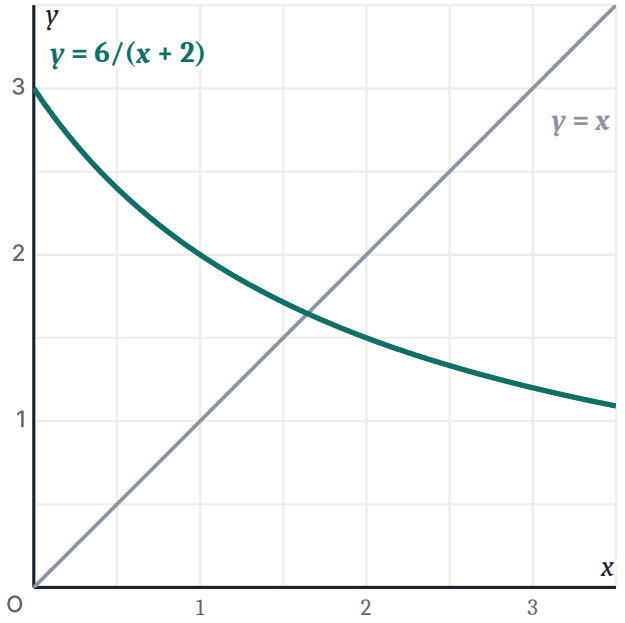
3 The graphs of $y = x$ and $y = \sqrt{5x + 6}$ are shown. Use $x_{n+1} = \sqrt{5x_n + 6}$ with $x_0 = 1$ to find x_1 and x_2. On the graph, draw the diagram and mark x_1 and x_2. Which root does it approach?

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4 The graphs of $y = x$ and $y = \dfrac{6}{x + 2}$ are shown. Use $x_{n+1} = \dfrac{6}{x_n + 2}$ with $x_0 = 0$ to find x_1, x_2, x_3. On the graph, draw the diagram and mark them. Describe it.

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5 $e^x - 2x - 1 = 0$ has roots 0 and 1.2564.

(a) Use $x_{n+1} = \ln(2x_n + 1)$ with $x_0 = 1$ to find x_1, x_2, x_3.

(b) Use $x_{n+1} = \dfrac{e^{x_n} - 1}{2}$ with $x_0 = 1$ to find x_1, x_2, x_3.

(c) Which root does each approach? Is each diagram a staircase or a cobweb?

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6 $x^2 + x - 6 = 0$ has roots 2 and −3.

(a) Use $x_{n+1} = 6 - x_n^2$ with $x_0 = 1.9$ to find x_1, x_2, x_3. Describe the diagram.

(b) Show that the equation can be written as $x = \sqrt{6 - x}$. Use $x_{n+1} = \sqrt{6 - x_n}$ with $x_0 = 1.9$ to find x_1, x_2, x_3. Describe the diagram.

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7 $x_{n+1} = 1 + \dfrac{1}{2}\sin x_n$ (x_n in radians).

(a) With $x_0 = 1$, find x_1, x_2, x_3. Staircase or cobweb?

(b) The limit α is a root of $f(x) = 2x - 2 - \sin x = 0$. Show that $\alpha = 1.499$ correct to 3 d.p.

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8 $x^2 - 7x + 6 = 0$ has roots 1 and 6.

(a) Use $x_{n+1} = \dfrac{x_n^2 + 6}{7}$ with $x_0 = 5$ to find x_1, x_2, x_3. Which root does it approach?

(b) Explain, using a sketch, why $x_{n+1} = \dfrac{x_n^2 + 6}{7}$ cannot be used to find the root 6.

(c) Show that the equation can be written as $x = \sqrt{7x - 6}$ and find x_1, x_2, x_3 from $x_0 = 5$.

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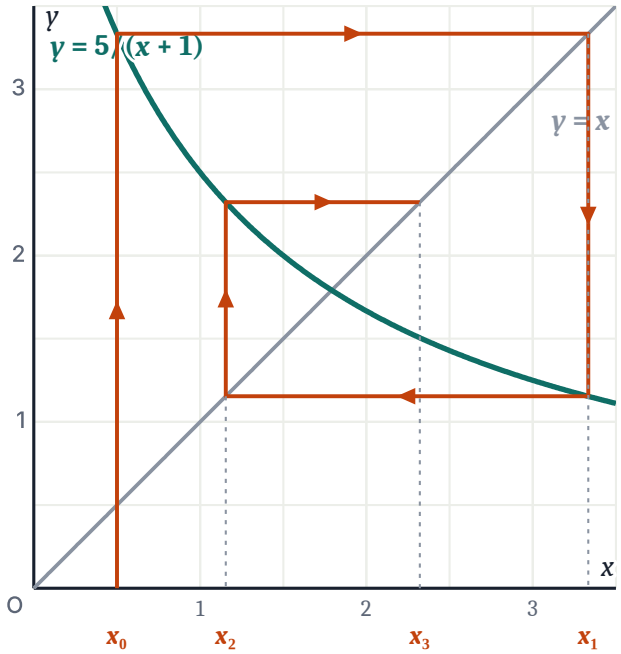
9 The diagram shows the graphs of $y = x$ and $y = \dfrac{5}{x+1}$ with the path of an iteration from $x_0 = 0.5$.

(a) Is it a staircase or a cobweb? Does it converge?

(b) Calculate x_1, x_2, x_3 and check them against the diagram.

(c) The path closes in on the root α of an equation $x^2 + bx + c = 0$. Find b and c.

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10 $f(x) = x^3 + 2x - 7$.

(a) Show that $f(x) = 0$ can be written as $x = \sqrt[3]{7 - 2x}$.

(b) Use $x_{n+1} = \sqrt[3]{7 - 2x_n}$ with $x_0 = 0$ to find x_1, x_2, x_3. Staircase or cobweb?

(c) Show that $\alpha = 1.569$ correct to 3 d.p.

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11 *AQA-style.* $f(x) = x^3 + 3x - 5$.

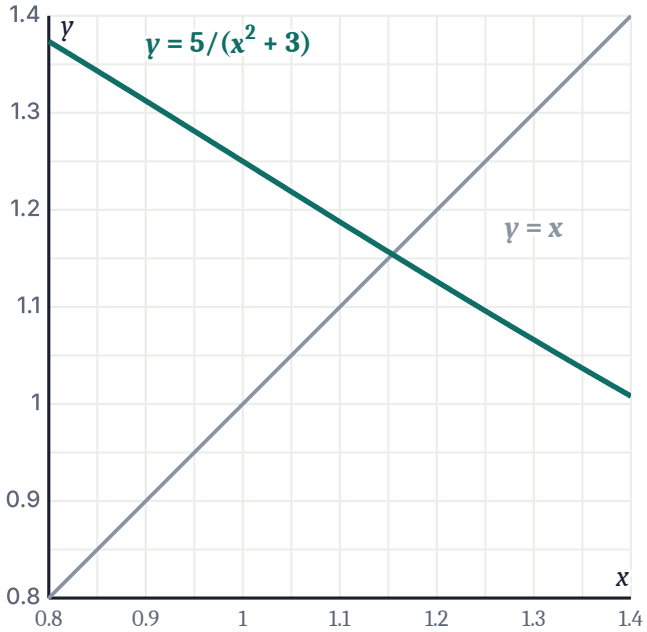
(a) Show that $f(x) = 0$ has a root α in the interval $[1.1, 1.2]$. [2 marks]

(b) Show that $f(x) = 0$ can be rearranged into the form $x = \dfrac{5}{x^2 + 3}$. [1 mark]

(c) Use $x_{n+1} = \dfrac{5}{x_n{}^2 + 3}$ with $x_1 = 1$ to find x_2 and x_3, giving your answers to three decimal places. [2 marks]

(d) The figure shows $y = x$ and $y = \dfrac{5}{x^2 + 3}$. On it, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. [2 marks]

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12 *Edexcel-style.* $f(x) = \ln(x + 3) - x + 1$, $x > -3$. The equation $f(x) = 0$ has a root α with $\alpha > 0$.

(a) Show that $f(x) = 0$ can be written as $x = 1 + \ln(x + 3)$. [1 mark]

(b) Use $x_{n+1} = 1 + \ln(x_n + 3)$ with $x_0 = 2$ to find x_1, x_2 and x_3 to 4 decimal places. [3 marks]

(c) By choosing a suitable interval, show that $\alpha = 2.749$ correct to 3 decimal places. [3 marks]

(d) State whether the iterates would form a staircase or a cobweb diagram, giving a reason. [1 mark]

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