The Newton–Raphson Method

Specification: AQA 7357: I2 · Edexcel 9MA0: 9.3

HOMEWORK Consolidation

1 $f(x) = x^3 + x^2 - 3x - 4$.

(a) Show that $f(x) = 0$ has a root in $[1.8, 1.9]$.

(b) Using $x_0 = 2$, apply the Newton–Raphson method twice to find x_1 and x_2 to 4 d.p.

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2 $f(x) = e^{-x} - \frac{x}{2}$.

(a) Write down the Newton–Raphson recurrence for f.

(b) Using $x_0 = 1$, find x_1, x_2 and x_3 to 4 d.p.

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3 $f(x) = 2\ln x - x + 2$, $x > 0$, has roots near 0.46 and 5.36.

(a) Using $x_0 = 5$, find x_1 and x_2 to 4 d.p.

(b) Explain why $x_0 = 2$ cannot be used.

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4 $f(x) = \sin 2x - \frac{x}{2}$ (x in radians).

(a) Using $x_0 = 1.2$, find x_1 and x_2 to 4 d.p.

(b) Show that the root $\alpha = 1.237$ correct to 3 d.p.

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5 $f(x) = x^3 - 6x^2 + 9x - 2$.

(a) Find the x-coordinates of the stationary points. Explain why $x_0 = 3$ cannot be used.

(b) Using $x_0 = 4$, find x_1, x_2 and x_3 to 4 d.p.

(c) Show that $f(x) = (x - 2)(x^2 - 4x + 1)$. Find the exact root the iterates approach.

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6 No calculator. $f(x) = x^3 - 18x + 54$.

(a) Using $x_0 = 3$, find x_1 and x_2.

(b) Describe what happens if the method is continued.

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7 Use the Newton–Raphson method with $x_0 = 1.5$ to solve $3^x = x + 4$. Find x_1 and x_2 to 4 d.p.

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8 (a) Show that the Newton–Raphson recurrence for $f(x) = x^3 - 20$ can be written as $x_{n+1} = \frac{2x_n^{\,3} + 20}{3x_n^{\,2}}$.

(b) Using $x_0 = 3$, find x_1, x_2, x_3 to 4 d.p. and compare with $\sqrt[3]{20}$.

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9 $x^3 + x - 3 = 0$ has one root α (Lesson 3).

(a) Using the Newton–Raphson method with $x_0 = 1$, find x_1, x_2, x_3 to 4 d.p.

(b) Using $x_{n+1} = \sqrt[3]{3 - x_n}$ with $x_0 = 1$, find x_1, x_2, x_3 to 4 d.p.

(c) Which method gets closer to α in three steps?

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10 $f(x) = x^3 + kx - 2$, where k is a constant. A pupil uses the Newton–Raphson method with $x_0 = 2$ and finds $x_1 = 2.25$. Also $f(2) = -2$.

(a) Use the Newton–Raphson formula to find $f'(2)$.

(b) Find k in two ways.

(c) Find x_2 to 4 d.p.

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11 *AQA-style.* $f(x) = x^4 + x - 4$. The equation $f(x) = 0$ has a positive root α.

(a) Show that α lies between 1.2 and 1.3. [2 marks]

(b) Taking $x_1 = 1.5$ as a first approximation, use the Newton–Raphson method to find x_2 and x_3, giving your answers to three decimal places. [3 marks]

(c) Explain why $x_1 = -0.63$ would not be a suitable first approximation. [2 marks]

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12 *Edexcel-style.* $f(x) = x^2 + 2\ln x - 5$, $x > 0$.

(a) Show that $f(x) = 0$ has a root α in the interval $[1.9, 2]$. [2 marks]

(b) Find $f'(x)$. [2 marks]

(c) Taking $x_0 = 2$ as a first approximation to α, apply the Newton–Raphson procedure once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places. [2 marks]

(d) By considering the change of sign of $f(x)$ over an appropriate interval, show that $\alpha = 1.922$ correct to 3 decimal places. [2 marks]

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