Numerical Methods in Context

Specification: AQA 7357: I4 (with I1, I2) · Edexcel 9MA0: 9.5 (with 9.1–9.3)

HOMEWORK Consolidation

1 The value of a tablet, £V, t years after it is bought, is $V = 600(0.8)^t - 30t$.
(a) Find its value after 2 years.
(b) Show that it is worthless between 5 and 6 years.
(c) Find $V'(t)$. Using $t_0 = 6$, apply the Newton–Raphson method once (3 d.p.).
(d) Give t_1 in years and months.
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2 The temperature in a greenhouse, T °C, t hours after 6 am, is $T = 18 + 8\sin(0.3t) - 0.6t$. A heater comes on when it falls to 15 °C.
(a) Find the temperature at 8 am.
(b) Show that $T = 15$ between $t = 9$ and $t = 10$.
(c) Using $t_0 = 9$, apply the Newton–Raphson method twice to $f(t) = T - 15$ (4 d.p.).
(d) At what time does the heater come on?
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3 A second medicine has concentration $C = 8te^{-0.5t}$ mg/L, t hours after it is taken. It works while $C \geq 3$.
(a) Write $f(t) = C - 3$ and find $f'(t)$.
(b) Using $t_0 = 0.5$ and then $t_0 = 5.5$, apply the Newton–Raphson method twice each time (4 d.p.).
(c) For how long does the medicine work, to the nearest minute?
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4 The height h m of a firework rocket, t seconds after it leaves the ground, is modelled by $h = 20\sin(0.1t) + 3t - 0.25t^2$ (t in radians).
(a) Find $h'(t)$. Show that the greatest height occurs between $t = 8$ and $t = 9$.
(b) Show that $h'(t) = 0$ can be written as $t = 4\cos(0.1t) + 6$.
(c) Using $t_0 = 8$, find t_1, t_2 and t_3 (3 d.p.).
(d) The iterates approach $t = 8.61$. Find the greatest height (3 s.f.).
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5 A paper plane's height is $h = 30\sin(0.1t) + t - 0.25t^2$ m, t seconds after launch (t in radians).
(a) Show that, for $t > 0$, $h = 0$ can be written as $t = \sqrt{120\sin(0.1t) + 4t}$.
(b) Using $t_0 = 15$, find t_1, t_2, t_3 (3 d.p.).
(c) The iterates approach 12.93. State a suitable domain for the model and find $h(15)$.
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6 A population of rare birds is modelled by $P = 250 + 80\sin(0.5t) - 12t$, t months after a survey.
(a) Find $P(10)$ and $P(20)$.
(b) Show that the model gives $P = 0$ between $t = 19$ and $t = 20$.
(c) Criticise the model and suggest a range of validity.
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7 The height h m of a quadcopter, t seconds after launch, is $h = 3 + 18\sin(0.25t) - 0.15t^2$ (t in radians).
(a) Show that $h'(t) = 0$ can be written as $t = 15\cos(0.25t)$.
(b) Using $t_0 = 5$, find t_1, t_2, t_3 with this formula (3 d.p.). What happens?
(c) Show that $h'(t) = 0$ can also be written as $t = 4\arccos(t/15)$. Using $t_0 = 5$, find t_1, t_2, t_3.
(d) The second formula converges to $t = 4.94$. Find the greatest height (3 s.f.).
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8 The number of people at a fair, N thousand, t hours after it opens at 9 am, is $N = 3te^{-0.2t}$. The car park is full while $N \geq 4$.
(a) Write $f(t) = N - 4$ and find $f'(t)$.
(b) Explain why $t_0 = 5$ cannot be used to find when the car park stops being full.
(c) Using $t_0 = 9$, apply the Newton–Raphson method twice (4 d.p.).
(d) At what time does the car park stop being full?
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9 A company spends £x thousand on a campaign. Its profit is £P thousand, where $P = 10\ln(x+1) - x - 4$, $x \geq 0$.

(a) Show that the profit is zero for some x between 30 and 31.

(b) Using $x_0 = 30$, apply the Newton–Raphson method once (3 d.p.).

(c) Find the exact spending that gives the greatest profit, and that profit to the nearest £.

(d) What advice would you give the company?

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10 Write each answer in context.

(a) The car in You do Q7 is worth £5000 when $t = 3.9189$ years. Give this in years and months.

(b) The oven in Spot the error Q2 reaches 180 °C when $t = 1.0748$ hours after 7 am. Give the clock time.

(c) The car model gives $V(4) = 4.8339$ (£ thousand). Give the value to the nearest £10.

AQA I4 · Edexcel 9.5

11 *AQA-style*. The number of visitors to a gallery, v thousand per month, x months after 1 January 2020, is modelled by

$$v(x) = 4\mathrm{e}^{-x} + 3\sin\left(\frac{x}{2}\right) + 0.4x,\ 0 \leq x \leq 10.$$

(a) Find $v'(x)$. [2 marks]

(b) Show that the x-coordinates of the turning points of $y = v(x)$ with $0 \leq x \leq 2\pi$ satisfy $x = 2\arccos\left(\dfrac{8\mathrm{e}^{-x} - 0.8}{3}\right)$. [2 marks]

(c) Using $x_{n+1} = 2\arccos\left(\dfrac{8\mathrm{e}^{-x_n} - 0.8}{3}\right)$ with $x_0 = 3$, find x_1, x_2 and x_3 to three decimal places. [3 marks]

(d) The manager claims that visitor numbers rose throughout March, April and May 2020. State, with a reason, whether the model supports the claim. [2 marks]

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12 *Edexcel-style*. The cross-section of a stream is modelled by $y = (x - 6)\ln(x+1)$, $0 \leq x \leq 6$, where x m is the distance from one bank and y m is the height relative to the water surface ($y < 0$ is under water).

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2 marks]

(b) Show that the deepest point lies between $x = 2.2$ and $x = 2.3$. [2 marks]

(c) Show that the x-coordinate of the deepest point satisfies $x = \dfrac{7}{1 + \ln(x+1)} - 1$. [2 marks]

(d) Using $x_{n+1} = \dfrac{7}{1 + \ln(x_n + 1)} - 1$ with $x_0 = 2$, find x_1, x_2 and x_3 to 3 decimal places. The iterates approach 2.22. Find the greatest depth of the stream to 2 decimal places. [3 marks]

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