Numerical Methods in Context

Specification: AQA 7357: I4 (with I1, I2) · Edexcel 9MA0: 9.5 (with 9.1–9.3)

INDEPENDENT

You do

A · Using a model

Answer in context, with units.

1 The value £V of a laptop t years after it is bought is $V = 1200(0.7)^t - 60t$.
(a) Find its value after 3 years.
(b) Show that the model predicts it is worthless between 4 and 5 years.

2 The depth of water is $D = 5 + 2\sin(0.5t) + 0.1t$ m, t hours after midnight.
(a) Find the depth at 3 am.
(b) Show that the depth first falls to 4 m between 8 am and 9 am.

3 A herd of deer has $P = 300 + 120\sin(0.4t) - 15t$ animals, t months after a count.
(a) Find the number of deer after 2 months.
(b) Show that the herd falls to 200 between $t = 7$ and $t = 8$.

4 An engine is switched off. Its temperature is $T = 20 + 70\mathrm{e}^{-0.2t} + 0.8t$ °C, t minutes later.
(a) Find its temperature after 5 minutes.
(b) Show that it cools to 40 °C between $t = 8$ and $t = 9$.

5 A shop spends £x thousand on advertising. Its profit is £P thousand, where $P = 12\ln(x + 1) - 1.5x - 2$, $x \geq 0$.
(a) Find P when $x = 0$. What does it mean?
(b) Show that the shop breaks even ($P = 0$) for some x between 24 and 25.

6 The medicine in Worked example 5: $C = 6t\mathrm{e}^{-0.5t}$ mg/L.
(a) Complete the table (2 d.p.).
(b) Between which whole numbers of hours does C rise past 2 mg/L, and fall back below it?

t	0	1	2	3	4	5	6	7
C								

B · Setting up

Write the equation first, then use a method.

7 A car is worth £V thousand, where $V = 16(0.82)^t - 0.6t$, t years after it is bought.
(a) Write an equation $f(t) = 0$ for the time when it is worth £5000.
(b) Find $f'(t)$.
(c) Using $t_0 = 4$, apply the Newton–Raphson method once (3 d.p.). Give the answer in years and months.

8 The engine in Q4: $T = 20 + 70\mathrm{e}^{-0.2t} + 0.8t$.
(a) Find $T'(t)$.
(b) Find the exact time when the temperature is lowest, and the lowest temperature (1 d.p.).
(c) Was a numerical method needed? Explain.

9 The medicine in Worked example 5: $C = 6t\mathrm{e}^{-0.5t}$.
(a) Show that $C = 2$ can be written as $t = 2\ln 3t$.
(b) Using $t_0 = 5$, find t_1, t_2, t_3 (3 d.p.).

10 The drone in Worked example 2: $h = 30\sin(0.1t) + 2t - 0.2t^2$ (t in radians). Write down the Newton–Raphson recurrence for $h(t) = 0$. Use it with $t_0 = 18$ to find t_1 (4 d.p.). What does t_1 estimate?

C · Reasoning

Explain in context.

11 The deer in Q3: $P = 300 + 120\sin(0.4t) - 15t$.
(a) Find P when $t = 12.5$ and when $t = 24$.
(b) Criticise the model.
(c) The model first gives $P = 0$ at $t = 12.04$. Suggest a range of validity.

12 The shop in Q5: $P = 12\ln(x + 1) - 1.5x - 2$ (£ thousand). $P = 0$ when $x = 0.213$ and when $x = 24.611$.
(a) What does each value mean?
(b) Find the exact spending that gives the greatest profit, and that profit to the nearest £.

13 The medicine in Worked example 5 ($f(t) = C - 2$). A pupil uses Newton–Raphson with $t_0 = 2$ to find when $C = 2$.
(a) Explain why $t_0 = 2$ fails.
(b) Another pupil uses $t_0 = 2.2$. Find t_1 and explain why it is not useful.
(c) Suggest a good t_0 for each time.

14 The depth model in Q2: $D = 5 + 2\sin(0.5t) + 0.1t$. The depth is 4 m when $t = 8.6996$ and $t = 9.7447$.
(a) Give these as clock times.
(b) For how long is the depth under 4 m?
(c) A sailor says the water is lowest at 9 am. Use $D'(9)$ and $D'(9.5)$ to comment.

D · Show that

15 The value of a phone, £V, t years after it is bought, is $V = 900(0.75)^t - 40t$.

(a) Show that it is worthless between 5 and 6 years.

(b) Find $V'(t)$. Using $t_0 = 5$, apply the Newton–Raphson method twice (4 d.p.).

(c) Show that this time is 5.14 years to 2 d.p.

(d) Give it in years and months.

16 The cross-section of a channel is $y = (x - 8)\ln(0.5x + 1)$, $0 \le x \le 8$: x m from one bank, y m from the water surface ($y < 0$ is under water).

(a) Find y'. Show that the deepest point is between $x = 3.1$ and $x = 3.2$.

(b) Show that $y' = 0$ can be written as $x = \dfrac{10}{1 + \ln(0.5x + 1)} - 2$.

(c) Using $x_0 = 3$, find x_1, x_2, x_3 (3 d.p.).

(d) The iterates approach 3.14. Find the greatest depth (2 d.p.).

17 The depth of water at a ferry port, t hours after noon, is $D = 6 + 2.4\sin(0.5t) - 0.2t$ m, $0 \le t \le 12$. The ferry needs 5 m.

(a) Show that the depth first falls to 5 m between $t = 6$ and $t = 7$.

(b) Using $t_0 = 6$, apply the Newton–Raphson method once to $f(t) = D - 5$ (4 d.p.).

(c) Show that this time is $t = 6.10$ to 2 d.p.

(d) Give the clock time, to the nearest minute.

E · Exam-style question AQA-style. Use the mark scheme to self-assess.

18 A ball is thrown from ground level. Its height h metres, t seconds later, is modelled by

$h(t) = 20\sin\left(\frac{t}{5}\right) - 4\cos\left(\frac{t}{5}\right) + 4 - 0.5t^2$, $t \ge 0$.

The ball lands when $t = T$.

(a) Show that T is a solution of $t = \sqrt{8 + 40\sin\left(\frac{t}{5}\right) - 8\cos\left(\frac{t}{5}\right)}$. [2 marks]

(b) Using $t_{n+1} = \sqrt{8 + 40\sin\left(\frac{t_n}{5}\right) - 8\cos\left(\frac{t_n}{5}\right)}$ with $t_0 = 6$, find t_1, t_2 and t_3, giving your answers to three decimal places. [3 marks]

(c) Find $h'(t)$. Taking 6 as a first approximation, apply the Newton–Raphson method once to obtain a second approximation for T. Give your answer to three decimal places. [3 marks]