The Trapezium Rule

Specification: AQA 7357: I3 (with H4, I4) · Edexcel 9MA0: 9.4 (with 9.5)

HOMEWORK Consolidation

1 (a) $\int_2^5 f(x)\,\mathrm{d}x$ is estimated with 6 strips. Write down h and the x-values.
(b) $\int_0^2 g(x)\,\mathrm{d}x$ is estimated using nine ordinates. How many strips? What is h?
AQA I3 · Edexcel 9.4

2 Use the trapezium rule with 4 strips to estimate $\int_0^2 \mathrm{e}^{\sqrt{x}}\,\mathrm{d}x$. Give your answer to 3 d.p.
AQA I3 · Edexcel 9.4

3 The table shows some values of $y = \sqrt{\sin x}$ (x in radians).
(a) Complete the table (4 d.p.).
(b) Use the trapezium rule with all the values to estimate $\int_0^{\pi} \sqrt{\sin x}\,\mathrm{d}x$ (3 s.f.).
(c) The curve is concave on $0 < x < \pi$. Is your estimate an over- or underestimate?

x	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
y	0			1	0.9306	0.7071	0

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4 (a) Use the trapezium rule with 4 strips to estimate $\int_0^1 \ln(1+x^2)\,\mathrm{d}x$ (3 d.p.).
(b) Given that $f''(x) = \dfrac{2(1-x^2)}{(1+x^2)^2}$, state whether (a) is an over- or underestimate, with a reason.
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5 (a) Find the exact value of $\int_1^4 \left(x + \dfrac{4}{x^2}\right)\mathrm{d}x$.
(b) Use the trapezium rule with 3 strips (4 s.f.).
(c) Find the percentage error. Explain why your estimate is too big.
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6 $y = \mathrm{e}^{-x}$, $0 \le x \le 2$. Let $I = \int_0^2 \mathrm{e}^{-x}\,\mathrm{d}x$.
(a) Using 4 strips, find the lower and upper rectangle sums.
(b) Use the trapezium rule with 4 strips.
(c) The curve is convex. Give the closest limits that I must lie between (3 d.p.). Check them with the exact value $1 - \mathrm{e}^{-2}$.
AQA I3 · Edexcel 9.4 · AQA H4

7 (a) Use the trapezium rule with 4 strips to estimate $\int_0^1 \sqrt{1+x^4}\,\mathrm{d}x$ (3 d.p.).
(b) Hence estimate $\int_0^1 \sqrt{4+4x^4}\,\mathrm{d}x$.
(c) Hence estimate $\int_0^1 \left(\sqrt{1+x^4} + 5x^4\right)\mathrm{d}x$.
AQA I3 · Edexcel 9.4

8 A weather balloon is released. Its upward speed v m/s, t seconds later, is shown.

t	0	10	20	30	40	50	60
v	0	1.8	3.1	3.9	4.3	4.5	4.6

(a) Estimate the height it reaches after 60 seconds.
(b) The graph of v against t is concave. Is (a) too big or too small?
AQA I3 · Edexcel 9.4 · AQA I4 · Edexcel 9.5

9 A runner's speed is $v = 5\ln(t+1)$ m/s, $0 \le t \le 8$.
(a) Use the trapezium rule with 4 strips to estimate the distance run in the first 8 s (3 s.f.).
(b) Is this an over- or underestimate? Give a reason.
AQA I3 · Edexcel 9.4 · AQA I4 · Edexcel 9.5

10 Priya says: "If you double the number of strips, the error in the trapezium rule halves."
Test her claim with $\int_0^3 x^2\,\mathrm{d}x$, using 3 strips and then 6 strips.
AQA I3 · Edexcel 9.4

11 *AQA-style*. A particle moves in a straight line. Its velocity v m/s at time t seconds is $v = 12te^{-0.4t}$, $0 \le t \le 10$.

(a) Use the trapezium rule with six ordinates to estimate the distance travelled in the 10 seconds, to 3 significant figures. [3 marks]

(b) Find $v'(t)$. Hence find the greatest velocity, to 3 significant figures. [3 marks]

(c) Given that $v''(t) = 0$ when $t = 5$, explain why it cannot be decided from the shape whether (a) is an overestimate or an underestimate. [2 marks]

(d) State how a more accurate estimate could be found with the trapezium rule. [1 mark]

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12 *Edexcel-style*. The table shows some values of $y = \sqrt{x}\ \ln x$.

x	1	1.5	2	2.5	3
y	0		0.9803		1.9029

(a) Complete the table, giving the values to 4 decimal places. [1 mark]

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for $\int_1^3 \sqrt{x}\ln x\,\mathrm{d}x$, giving your answer to 3 significant figures. [4 marks]

(c) Using your estimate, find approximate values for (i) $\int_1^3 \sqrt{x}\ \ln(x^2)\,\mathrm{d}x$ (ii) $\int_1^3 (\sqrt{x}\ln x + 3)\,\mathrm{d}x$. [2 marks]

(d) State how your approximation could be improved. [1 mark]

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