The Trapezium Rule

Specification: AQA 7357: I3 (with H4, I4) · Edexcel 9MA0: 9.4 (with 9.5)

INDEPENDENT **You do**

A · Using the rule Find h first. Ends once, middle twice.

1 (a) $\int_1^4 f(x)\,dx$ is estimated with 6 strips. Write down h and the x-values.
(b) $\int_0^3 g(x)\,dx$ is estimated using seven ordinates. How many strips are there? Write down h and the x-values.

2 Use the trapezium rule with 4 strips to estimate $\int_0^2 \sqrt{4+x^2}\,dx$. Give your answer to 3 d.p.

3 Use the trapezium rule with five ordinates to estimate $\int_0^1 \frac{1}{1+x^3}\,dx$. Give your answer to 4 s.f.

4 Use the trapezium rule with 4 strips to estimate $\int_0^1 \sin(x^2)\,dx$ (x in radians). Give your answer to 3 s.f.

5 The table shows some values of $y = \ln(1+\sqrt{x})$.
(a) Complete the table (4 d.p.).
(b) Use the trapezium rule with all the values to estimate $\int_0^3 \ln(1+\sqrt{x})\,dx$ (3 s.f.).

x	0	0.5	1	1.5	2	2.5	3
y	0	0.5348	0.6931		0.8814		1.0051

6 (a) Complete the table for $y = \frac{1}{\sqrt{1+x^2}}$ (4 d.p.).
(b) Use the trapezium rule with all the values to estimate $\int_0^2 \frac{1}{\sqrt{1+x^2}}\,dx$ (3 d.p.).

x	0	0.5	1	1.5	2
y					

B · Over or under? Better estimates Decide from the shape: f″(x).

7 (a) Find the exact value of $\int_1^4 \frac{1}{\sqrt{x}}\,dx$.
(b) Use the trapezium rule with 3 strips to estimate it (3 s.f.).
(c) Find the percentage error (2 s.f.).
(d) Explain why it is an overestimate.

8 (a) Find the exact value of $\int_0^2 x^3\,dx$.
(b) Use the trapezium rule with 2 strips, and then with 4 strips.
(c) Find the error in each. What happens to the error when the number of strips doubles?
(d) Explain why both are overestimates.

9 (a) Explain why $\int_0^3 \sqrt{9-x^2}\,dx = \frac{9\pi}{4}$.
(b) Use the trapezium rule with 3 strips to estimate it (3 s.f.).
(c) Find the percentage error.
(d) Is it an over- or underestimate? Why is the error so large?

10 $f(x) = x^3 - 3x^2 + 4$.
(a) Find $f''(x)$. Explain why you cannot tell from the shape whether the trapezium rule over- or underestimates $\int_0^3 (x^3 - 3x^2 + 4)\,dx$.
(b) Use the trapezium rule with 3 strips.
(c) Find the exact value. Was your estimate an over- or underestimate?

C · Reasoning Explain each conclusion.

11 $y = 2^x$, $0 \le x \le 2$. Let $I = \int_0^2 2^x\,dx$.
(a) Using 4 strips, find the lower and upper rectangle sums.
(b) Use the trapezium rule with 4 strips.
(c) The curve is convex. Give the closest limits that I must lie between (3 d.p.).

12 (a) Show that the trapezium rule with 4 strips gives $\int_0^2 \sqrt{1+x^4}\,dx \approx 3.734$.
(b) Hence estimate $\int_0^2 (5\sqrt{1+x^4} - 3x^2)\,dx$.
(c) Hence estimate $\int_0^2 \sqrt{9+9x^4}\,dx$.

13 (a) Use the trapezium rule with 2 strips, then 4 strips, to estimate $\int_0^4 x^2\,\mathrm{d}x$.
(b) Find the exact value, and the error in each estimate.
(c) Predict the estimate with 8 strips. Check your prediction.

14 (a) Use the trapezium rule with 4 strips to estimate $\int_0^4 (x^2 - 4x)\,\mathrm{d}x$.
(b) Find the exact value. Hence find the area between the curve and the x-axis.
(c) Is (a) an over- or underestimate of the integral? Of the area?

D · In context The area under a rate graph is a total: distance, volume.

15 A car accelerates from rest. Its speed v m/s after t seconds is shown.

t	0	5	10	15	20	25	30
v	0	8.2	13.6	17.1	19.2	20.3	20.8

(a) Estimate the distance it travels in the 30 seconds.
(b) The speed–time graph is concave. Is (a) too big or too small?

16 A train's speed is $v = 20(1 - \mathrm{e}^{-0.3t})$ m/s, $0 \le t \le 10$.
(a) Use the trapezium rule with 5 strips to estimate the distance it travels in the first 10 s (3 s.f.).
(b) Show that the graph of v is concave. What does this tell you about (a)?

17 Water flows into a tank. The rate r litres per minute, t minutes after a tap is opened, is shown.

t	0	1	2	3	4	5	6
r	12.0	11.2	10.1	8.8	7.2	5.4	3.3

(a) Estimate the volume of water that flows in during the 6 minutes (3 s.f.).
(b) The rate falls faster and faster (the graph is concave). Is (a) too big or too small?

E · Exam-style question AQA-style. Use the mark scheme to self-assess.

18 (a) Use the trapezium rule with five ordinates to find an approximate value for $\int_0^2 \frac{12}{x^2+4}\,\mathrm{d}x$, giving your answer to four significant figures. [3 marks]
(b) The curve $y = \frac{12}{x^2+4}$ has a point of inflection at $x = 2/\sqrt{3}$. Explain why you cannot tell from the shape of the curve whether your answer to (a) is an overestimate or an underestimate. [1 mark]
(c) The curve is decreasing for $x > 0$. By considering rectangles, find limits, to 3 s.f., that the value of the integral must lie between. [3 marks]
(d) State how the approximation in (a) could be improved. [1 mark]