Unit Review

Specification: AQA 7357: I1, I2, I3, I4 · Edexcel 9MA0: 9.1, 9.2, 9.3, 9.4, 9.5

HOMEWORK Consolidation

1 $f(x) = \ln x + x - 3$.

(a) Show that $f(x) = 0$ has a root α in the interval $[2.2, 2.3]$.

(b) Show that $\alpha = 2.21$ correct to 2 d.p.

AQA I1 · Edexcel 9.1

2 (a) Show that the equation $x + e^{-x} = 2$ has a root α in the interval $[1.8, 1.9]$.

(b) Use $x_{n+1} = 2 - e^{-x_n}$ with $x_0 = 1$ to find x_1, x_2, x_3 and x_4 to 4 d.p.

AQA I1 · Edexcel 9.1 · AQA I2 · Edexcel 9.2

3 $x^2 - 3x - 1 = 0$ has a root $\alpha = \dfrac{3 + \sqrt{13}}{2} \approx 3.303$.

Each rearrangement below is used with $x_0 = 3$. For each, find x_1, x_2 and x_3 to 4 d.p., and say whether the iterates converge to α and whether the diagram is a staircase or a cobweb.

(a) $x = 3 + \dfrac{1}{x}$

(b) $x = \sqrt{3x + 1}$

(c) $x = \dfrac{x^2 - 1}{3}$

AQA I1, G3 · Edexcel 9.1, 7.3

4 $f(x) = 2\sin x - x$ (x in radians).

(a) Find $f'(x)$.

(b) Use the Newton–Raphson method with $x_0 = 2$ to find x_1 and x_2 to 4 d.p.

(c) Explain why $x_0 = \pi/3$ would not be a suitable first approximation.

AQA I2 · Edexcel 9.3

5 The table shows some values of $y = \dfrac{1}{1 + \sqrt{x}}$.

x	0	1	2	3	4
y	1	0.5			

(a) Complete the table, giving values to 4 d.p.

(b) Use the trapezium rule with all the values to estimate $\int_0^4 \dfrac{1}{1 + \sqrt{x}}\,dx$, to 3 d.p.

(c) The exact value is $4 - 2\ln 3$. Find the percentage error, and say whether your estimate is an over- or underestimate.

AQA I3 · Edexcel 9.4

6 The value of a car is modelled by $V = 15\,000(0.8)^t - 500t$ pounds, t years after it was bought.

(a) Find its value after 2 years.

(b) Show that the car is worth £5000 at some time between $t = 3$ and $t = 4$.

(c) Use the Newton–Raphson method with $t_0 = 3.5$ to find t_1 and t_2. Hence find, to the nearest month, when it is worth £5000.

(d) Explain why the model is not valid for large values of t.

AQA I4 · Edexcel 9.5 · AQA I2 · Edexcel 9.3

7 Let $I = \int_1^3 \dfrac{4}{x^2}\,dx$.

(a) Using 4 strips, find the lower rectangle sum.

(b) Use the trapezium rule with 4 strips.

(c) The curve $y = \dfrac{4}{x^2}$ is convex. Give the closest limits that I must lie between (3 d.p.). Check them with the exact value $\dfrac{8}{3}$.

AQA I3 · Edexcel 9.4 · AQA H4

8 The height of a drone is modelled by $h = 20\sin(0.1t) + 4t - 0.25t^2$ metres, t seconds after take-off (angles in radians).

(a) Show that h is greatest when $t = 8 + 4\cos(0.1t)$.

(b) Use $t_{n+1} = 8 + 4\cos(0.1t_n)$ with $t_0 = 10$ to find t_1 to t_4 (4 d.p.).

(c) Show that the greatest height is reached at $t = 10.12$ to 2 d.p.

(d) Find the greatest height.

AQA I4 · Edexcel 9.5 · AQA I2 · Edexcel 9.2

9 $f(x) = \tan x - 2$ (x in radians).

(a) Find $f(1)$ and $f(2)$.

(b) Mia says: "These have the same sign, so $f(x) = 0$ has no root between 1 and 2." Show that she is wrong, and explain why the change-of-sign test misled her.

AQA I1 · Edexcel 9.1

10 $f(x) = e^x - 4x$.

(a) Show that $f(x) = 0$ has a root β in the interval $[2, 2.2]$.

(b) Using $x_{n+1} = \dfrac{e^{x_n}}{4}$ with $x_0 = 2.1$, find x_1, x_2 and x_3 to 4 d.p. What happens?

(c) Using $x_{n+1} = \ln(4x_n)$ with $x_0 = 2.1$, find x_1, x_2 and x_3 to 4 d.p. What happens?

AQA I1 · Edexcel 9.1 · AQA I1, G3 · Edexcel 9.1, 7.3

11 *AQA-style*. The concentration of a drug in a patient's blood is modelled by $C = 12te^{-0.3t}$ mg/L, t hours after it is given. The drug is effective while $C \geq 10$.

(a) Show that, as it falls, C reaches 10 between $t = 7$ and $t = 8$. [2 marks]

(b) Taking $t_1 = 7$, use the Newton–Raphson method to find t_2, giving your answer to four decimal places. [3 marks]

(c) C first reaches 10 when $t = 1.19$. Find, to the nearest minute, how long the drug is effective. [2 marks]

AQA I4 · Edexcel 9.5 · AQA I2 · Edexcel 9.3 · AQA-style

12 *Edexcel-style*. $f(x) = 2\ln x - x + 3$, $x > 0$.

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[6, 7]$. [2 marks]

(b) Using $x_{n+1} = 2\ln x_n + 3$ with $x_1 = 6$, find x_2, x_3 and x_4, giving each to 4 decimal places. [2 marks]

(c) State whether a diagram of these iterates would be a staircase or a cobweb, giving a reason. [1 mark]

(d) Use the trapezium rule with 4 strips to estimate $\int_1^5 (2\ln x - x + 3)\,dx$, giving your answer to 3 significant figures. [3 marks]

AQA I2 · Edexcel 9.2 · AQA I3 · Edexcel 9.4 · Edexcel-style