Unit Review

Specification: AQA 7357: I1, I2, I3, I4 · Edexcel 9MA0: 9.1, 9.2, 9.3, 9.4, 9.5

INDEPENDENT

Mixed practice

A · Fluency One method each. Which one?

1 $f(x) = 2x^3 - 3x - 4$. Show that $f(x) = 0$ has a root in the interval $[1.6, 1.7]$.

2 Use $x_{n+1} = \ln(3x_n + 2)$ with $x_0 = 2$ to find x_1, x_2, x_3 and x_4, giving each to 4 d.p.

3 $f(x) = x^3 + 2x - 7$. Use the Newton–Raphson method with $x_0 = 1.5$ to find x_1 and x_2, giving each to 4 d.p.

4 The table shows values of $y = f(x)$. Use the trapezium rule with all the values to estimate $\int_0^2 f(x)\,dx$. Give your answer to 3 d.p.

x	0	0.5	1	1.5	2
y	2	2.0310	2.2361	2.7157	3.4641

5 Without a calculator, use $x_{n+1} = \dfrac{6}{x_n + 1}$ with $x_0 = 1$ to find x_1 to x_4 as exact values.
(a) Is the diagram of the iterates a staircase or a cobweb?
(b) Find the limit α exactly.

6 The depth of water in a harbour is modelled by $d = 3 + 2\sin(0.5t) - 0.1t^2$ metres, t hours after midnight (angles in radians).
Show that the depth is 4 m at some time between 01:00 and 02:00.

B · Multi-step Several methods in one question.

7 (a) Show that $x^3 + 5x - 10 = 0$ has a root α in the interval $[1.4, 1.5]$.
(b) Show that the equation can be written as $x = \dfrac{10}{x^2 + 5}$.
(c) Use $x_{n+1} = \dfrac{10}{x_n^{\,2} + 5}$ with $x_0 = 1.4$ to find x_1, x_2 and x_3 to 4 d.p.
(d) Show that $\alpha = 1.42$ correct to 2 d.p.

8 $f(x) = x^2 - 3\cos x$ (x in radians).
(a) Find $f'(x)$.
(b) Use the Newton–Raphson method with $x_0 = 1.2$ to find x_1 and x_2 to 4 d.p.
(c) Show that the root of $f(x) = 0$ near 1.2 is 1.131 correct to 3 d.p.

9 (a) Find the exact value of $\int_1^3 (x^2 + 2x^{-2})\,dx$.
(b) Use the trapezium rule with 4 strips to estimate it, to 4 s.f.
(c) Find the percentage error of your estimate.
(d) Explain why your estimate is too big.

10 The concentration of a drug in the blood is modelled by $C = 8te^{-0.5t}$ mg/L, t hours after it is taken.
(a) Find the exact greatest concentration.
(b) The drug works while $C \geq 3$. Show that C falls to 3 between $t = 5$ and $t = 6$.
(c) Use the Newton–Raphson method with $t_0 = 5$ to find, to the nearest minute, when C falls to 3.

11 $x^2 + 2x - 4 = 0$ has a positive root $\alpha = \sqrt{5} - 1$.
(a) Use $x_{n+1} = \dfrac{4}{x_n + 2}$ with $x_0 = 1$ to find x_1 to x_4 as fractions. Staircase or cobweb?
(b) Use $x_{n+1} = \dfrac{4 - x_n^{\,2}}{2}$ with $x_0 = 1.2$ to find x_1 to x_4 (4 d.p.). Describe what happens.
(c) Explain, using the diagrams, why one iteration converges to α and the other does not.

C · Reasoning Explain. When does a method fail?

12 $f(x) = 4x^2 - 12x + 9$. Sam says: "$f(1) = 1$ and $f(2) = 1$ have the same sign, so $f(x) = 0$ has no root between 1 and 2."
Is Sam right? Explain, without a calculator.

13 $f(x) = e^x - 4x$.
(a) Explain why $x_0 = \ln 4$ cannot be used as a first approximation in the Newton–Raphson method.
(b) Find x_1 when $x_0 = 1.5$ and explain why it is a poor result.
(c) Starting from $x_0 = 2.5$, find x_1, x_2 and x_3 to 4 d.p.

14 Without a calculator. $\int_1^2 \frac{1}{x}\,\mathrm{d}x = \ln 2$.

(a) Using 2 strips, find the lower and upper rectangle sums and the trapezium rule estimate, as fractions.

(b) Using the shape of $y = \frac{1}{x}$, give the closest limits that $\ln 2$ must lie between.

D · Show that Write every line.

15 The curve $y = \mathrm{e}^x - x^3$ has a stationary point P with $0 < x < 1.5$.

(a) Show that the x-coordinate of P satisfies $x = \sqrt{\frac{\mathrm{e}^x}{3}}$.

(b) Use $x_{n+1} = \sqrt{\frac{\mathrm{e}^{x_n}}{3}}$ with $x_0 = 1$ to find x_1, x_2 and x_3 to 4 d.p.

(c) Show that the x-coordinate of P is 0.910 correct to 3 d.p.

16 $f(x) = x\mathrm{e}^x - 2$.

(a) Show that the Newton–Raphson recurrence for $f(x) = 0$ can be written as $x_{n+1} = \frac{x_n^{\,2} + 2\mathrm{e}^{-x_n}}{x_n + 1}$.

(b) Use it with $x_0 = 1$ to find x_1, x_2 and x_3 to 4 d.p.

E · Exam-style questions One AQA-style and one Edexcel-style question. Use the mark scheme to self-assess.

17 *AQA-style*. The curve $y = 3 + 2x - \mathrm{e}^x$ crosses the positive x-axis at $x = \alpha$.

(a) Show that α lies between 1.9 and 2.0. [2 marks]

(b) Taking $x_1 = 2$ as a first approximation for α, use the Newton–Raphson method to find x_2, giving your answer to four decimal places. [3 marks]

(c) Explain why $x_1 = \ln 2$ could not be used as a first approximation. [1 mark]

(d) Show that $\alpha = 1.92$ correct to two decimal places. [2 marks]

18 *Edexcel-style*. $f(x) = 4 - x^2 + \ln(x + 1)$, $x > -1$. The equation $f(x) = 0$ has a positive root α.

(a) Show that α lies in $[2.2, 2.3]$. [2 marks]

(b) Show that $f(x) = 0$ can be written as $x = \sqrt{4 + \ln(x + 1)}$. [1 mark]

(c) Using $x_{n+1} = \sqrt{4 + \ln(x_n + 1)}$ with $x_1 = 2$, find x_2, x_3 and x_4 to 4 decimal places. [2 marks]

(d) Use the trapezium rule with 4 strips to estimate $\int_0^2 f(x)\,\mathrm{d}x$ to 3 significant figures. [3 marks]

(e) Given that $f''(x) = -2 - 1/(x + 1)^2$, state, with a reason, whether your estimate is an overestimate or an underestimate. [1 mark]