Angles in All Four Quadrants and Exact Values

Specification: AQA 7357: E1, E3 · Edexcel 9MA0: 5.1, 5.3

INDEPENDENT

You do

A · Quadrants, signs and the axes No calculator.

1 State the quadrant of each angle θ and the signs of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

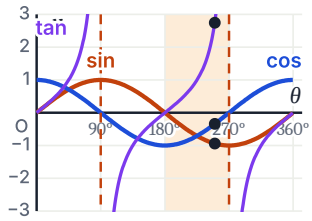
(a) 250°

(b) −30°

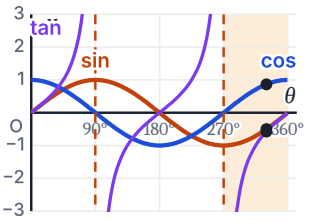
(c) 400°

(d) −250°

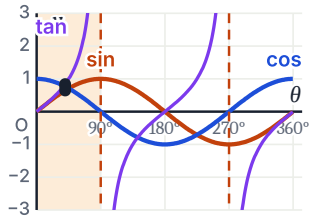
(a) 250°



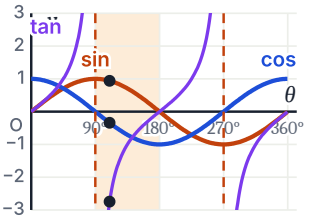
(b) −30° → 330°



(c) 400° → 40°



(d) −250° → 110°



sketch the three graphs

(a) 250° is between 180° and 270°: tan above the axis, sin and cos below. So sin −, cos −, tan +.

read the graphs: 180°–270°

(b) −30° + 360° = 330°: 330° is between 270° and 360°: cos above the axis, sin and tan below. So sin −, cos +, tan −.

read the graphs: 270°–360°

(c) 400° − 360° = 40°: 40° is between 0° and 90°: all three above the axis. So sin +, cos +, tan +. read the graphs: 0°–90°

(d) −250° + 360° = 110°: 110° is between 90° and 180°: sin above the axis, cos and tan below. So sin +, cos −, tan −.

read the graphs: 90°–180°

2 Positive or negative?

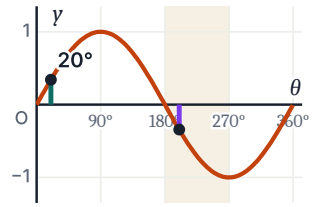
(a) $\sin 200°$

(b) $\cos 300°$

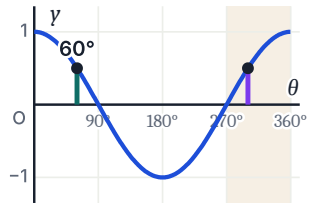
(c) $\tan 100°$

(d) $\cos(-120°)$

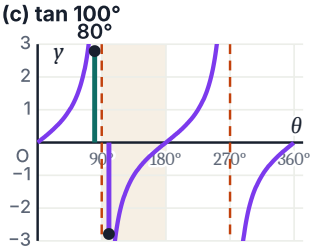
(a) sin 200°



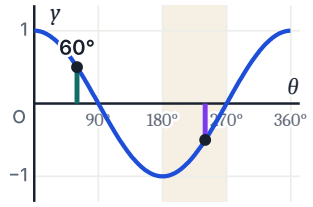
(b) cos 300°



(c) tan 100°



(d) cos(−120°) → 240°



sketch the graph

(a) 200° is between 180° and 270°, where the graph of $y = \sin\theta$ is below the axis: **negative**. graph below the axis

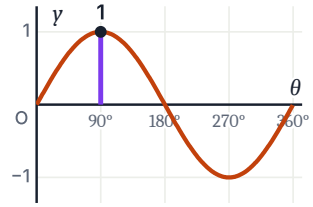
(b) 300° is between 270° and 360°, where the graph of $y = \cos\theta$ is above the axis: **positive**. graph above the axis

(c) 100° is between 90° and 180°, where the graph of $y = \tan\theta$ is below the axis: **negative**. graph below the axis

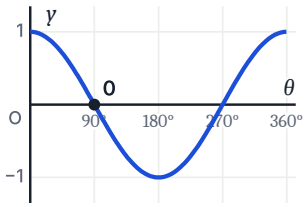
(d) $-120° + 360° = 240°$: 240° is between 180° and 270°, where the graph of $y = \cos\theta$ is below the axis: **negative**. graph below the axis

3 Write down the values of
(a) $\sin 450°$
(b) $\cos(-270°)$
(c) $\tan 540°$
(d) $\cos 900°$

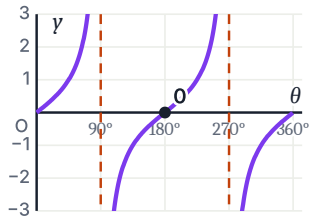
(a) sin 450° → 90°



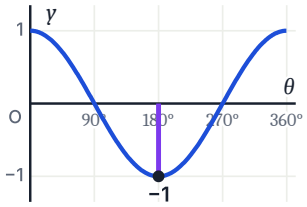
(b) cos(−270°) → 90°



(c) tan 540° → 180°



(d) cos 900° → 180°



sketch the graph

(a) $450° - 360° = 90°$: $y = \sin\theta$ has a maximum at 90°: $\sin 450° = 1$ — graph: maximum at 90°

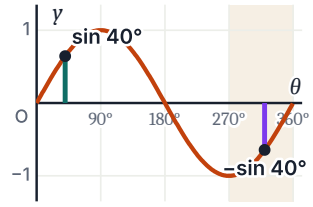
(b) $-270° + 360° = 90°$: $y = \cos\theta$ has a zero at 90°: $\cos(-270°) = 0$ — graph: zero at 90°

(c) $540° - 360° = 180°$: $y = \tan\theta$ has a zero at 180°: $\tan 540° = 0$ — graph: zero at 180°

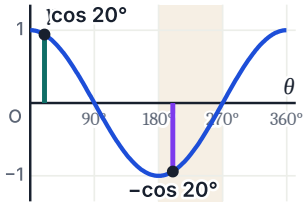
(d) $900° - 2 \times 360° = 180°$: $y = \cos\theta$ has a minimum at 180°: $\cos 900° = -1$ — graph: minimum at 180°

4 Write each one as a trigonometric ratio of an acute angle.
(a) $\sin 320°$
(b) $\cos 200°$
(c) $\tan 160°$
(d) $\sin(-240°)$

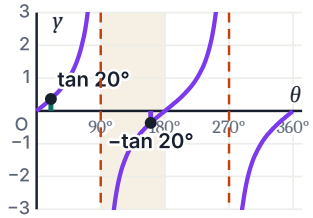
(a) sin 320°



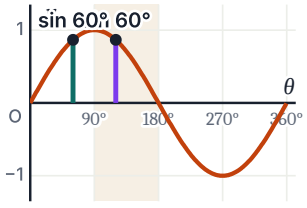
(b) cos 200°



(c) tan 160°



(d) sin(−240°) → 120°



sketch the graph

(a) $320° = 360° - 40°$, below the axis: $\sin 320° = -\sin 40°$ — graph: other side

(b) $200° = 180° + 20°$, below the axis: $\cos 200° = -\cos 20°$ — graph: other side

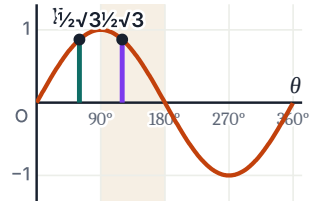
(c) $160° = 180° - 20°$, below the axis: $\tan 160° = -\tan 20°$ — graph: other side

(d) $-240° + 360° = 120°$: $120° = 180° - 60°$, above the axis: $\sin(-240°) = \sin 60°$ — graph: same side

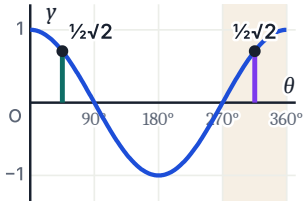
B · Exact values No calculator.

5 Find the exact values of
(a) sin 120°
(b) cos 315°
(c) tan 240°
(d) sin 330°

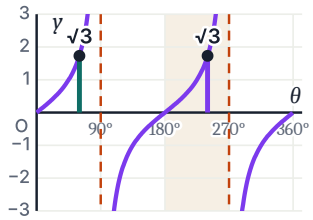
(a) sin 120°



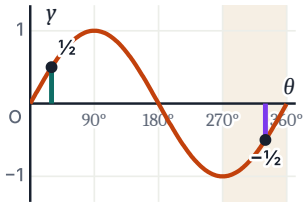
(b) cos 315°



(c) tan 240°



(d) sin 330°



sketch the graph

(a) 120° = 180° − 60°, above the axis: $\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$ — graph: same side

(b) 315° = 360° − 45°, above the axis: $\cos 315° = \cos 45° = \frac{\sqrt{2}}{2}$ — graph: same side

(c) 240° = 180° + 60°, above the axis: $\tan 240° = \tan 60° = \sqrt{3}$ — graph: same side

(d) 330° = 360° − 30°, below the axis: $\sin 330° = -\sin 30° = -\frac{1}{2}$ — graph: other side

6 Find the exact values of

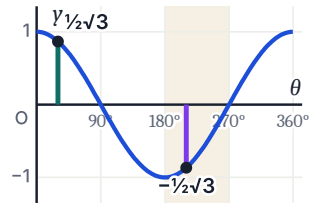
(a) $\cos 210°$

(b) $\tan(-135°)$

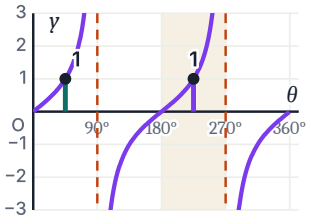
(c) $\sin 390°$

(d) $\cos 480°$

(a) cos 210°

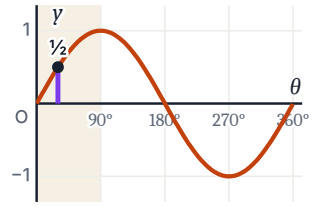


(b) tan(−135°) → 225°

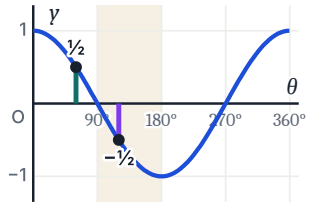


sketch the graph

(c) sin 390° → 30°



(d) cos 480° → 120°



(a) $210° = 180° + 30°$, below the axis: $\cos 210° = -\cos 30° = -\dfrac{\sqrt{3}}{2}$ — graph: other side

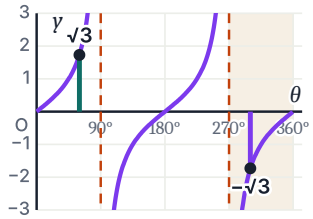
(b) $-135° + 360° = 225°$: $225° = 180° + 45°$, above the axis: $\tan(-135°) = \tan 45° = 1$ — graph: same side

(c) $390° - 360° = 30°$: $30°$ is already acute: $\sin 390° = \sin 30° = ½$ — graph: 0°–90°

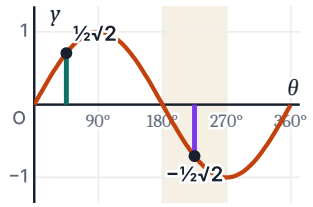
(d) $480° - 360° = 120°$: $120° = 180° - 60°$, below the axis: $\cos 480° = -\cos 60° = -½$ — graph: other side

7 Find the exact values of
(a) tan 300°
(b) sin 225°
(c) cos(−330°)
(d) tan 210°

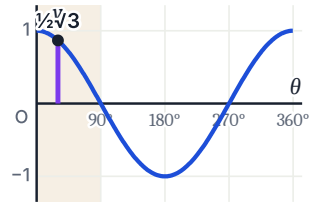
(a) tan 300°



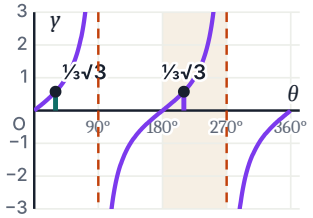
(b) sin 225°



(c) cos(−330°) → 30°



(d) tan 210°



sketch the graph

(a) 300° = 360° − 60°, below the axis: $\tan 300° = -\tan 60° = -\sqrt{3}$ — graph: other side

(b) 225° = 180° + 45°, below the axis: $\sin 225° = -\sin 45° = -\frac{\sqrt{2}}{2}$ — graph: other side

(c) −330° + 360° = 30°: 30° is already acute: $\cos(-330°) = \cos 30° = \frac{\sqrt{3}}{2}$ — graph: 0°–90°

(d) 210° = 180° + 30°, above the axis: $\tan 210° = \tan 30° = \frac{\sqrt{3}}{3}$ — graph: same side

8 Find the exact value of
(a) sin 150° + cos 240°
(b) tan 120° × tan 210°
(c) 2 cos 315°
(d) sin 60° ÷ cos 60°

(a) $\sin 150° = ½$, $\cos 240° = -½$: $½ + (-½) = 0$
exact, then add

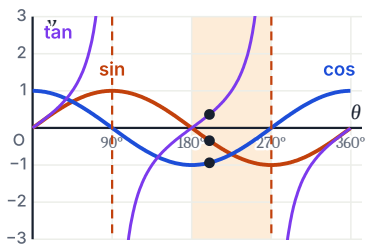
(b) $\tan 120° = -\sqrt{3}$, $\tan 210° = \frac{\sqrt{3}}{3}$: $-\sqrt{3} \times \frac{\sqrt{3}}{3} = -1$
exact, then multiply

(c) $\cos 315° = \frac{\sqrt{2}}{2}$: $2 \times \frac{\sqrt{2}}{2} = \sqrt{2}$ — exact, then × 2

(d) $\frac{\frac{\sqrt{3}}{2}}{½} = \sqrt{3}$ (that is tan 60°: Lesson 2) — ÷ ½ = × 2

C · Reasoning No calculator for this section.

9 An angle θ has $\sin\theta < 0$ and $\tan\theta > 0$.
(a) Which quadrant is θ in?
(b) Give one possible value of θ between 0° and 360°.



sketch the graphs

(a) The sine graph is below the axis on 180°–360°; the tan graph is above it on 0°–90° and 180°–270°.

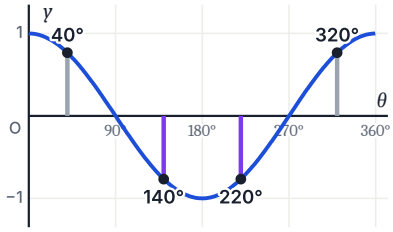
read both graphs

Both: between 180° and 270° (quadrant 3). 180°–270°

(b) Any angle from 180° to 270° (not the ends), e.g. 200°.

pick one inside

10 (a) List the four angles between 0° and 360° whose related acute angle is 40°.
(b) For which of them is $\cos\theta = -\cos 40°$?



sketch the graph

(a)
$40°, 180° - 40° = 140°, 180° + 40° = 220°, 360° - 40° = 320°$

α, 180° − α, 180° + α, 360° − α

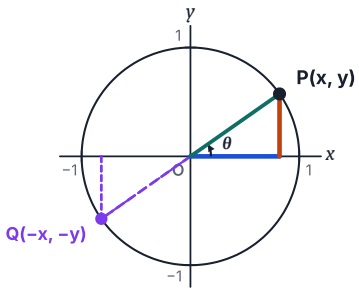
(b) The cosine graph is below the axis on 90°–270°: 140° and 220°

graph below the axis

11 P is on the unit circle at angle θ and Q is at angle $180° + \theta$.

(a) Explain why Q is the image of P under a half-turn about O.

(b) Hence show that $\sin(180° + \theta) = -\sin\theta$ and $\tan(180° + \theta) = \tan\theta$.



draw the unit circle

(a) OQ turns a further 180° from OP: it points the opposite way, so Q is P rotated 180° about O.

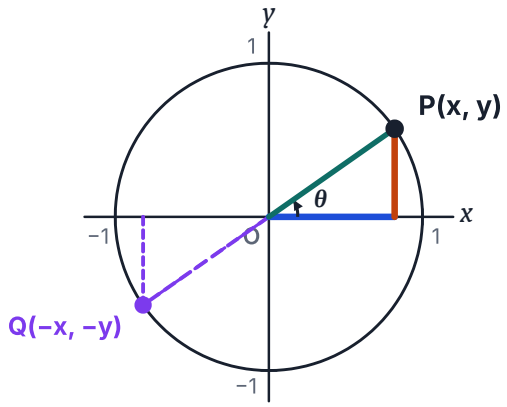
half-turn: + 180°

(b) A half-turn sends (x, y) to $(-x, -y)$: $Q = (-\cos\theta, -\sin\theta)$.

(x, y) → (−x, −y)

So $\sin(180° + \theta) = -\sin\theta$ and $\tan(180° + \theta) = \dfrac{-\sin\theta}{-\cos\theta} = \tan\theta$.

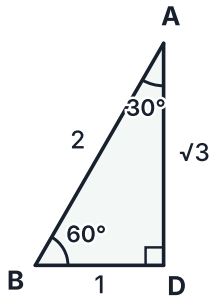
read the coordinates



Q = (−cos θ, −sin θ)

12 (a) Use the triangle with angles 30°, 60° and 90° to explain why $\sin 30° = \cos 60°$.

(b) Write $\cos 20°$ as the sine of an acute angle.



draw a diagram first

(a) The side opposite 30° is the side adjacent to 60°: both ratios are $\dfrac{1}{2}$.

same side, other angle

In general $\sin\theta = \cos(90° - \theta)$: the two acute angles of a right-angled triangle add up to 90°.

complementary angles

(b) $\cos 20° = \sin(90° - 20°) = \sin 70°$

90° − θ

D · Exact multi-step No calculator.

13 Show that $\sin 60° \cos 30° - \cos 60° \sin 30° = \sin 30°$.

LHS $= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - ½ \times ½$ — exact values

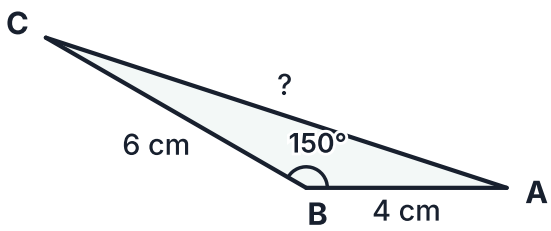
$= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = ½$ — simplify

$\sin 30° = ½$ = RHS — compare with the RHS

14 In triangle ABC, $AB = 4$ cm, $BC = 6$ cm and angle $ABC = 150°$.

(a) Find the exact area of the triangle.

(b) Find the exact value of AC^2.



draw a diagram first

(a) Area $= ½ \times 4 \times 6 \times \sin 150°$ — ½ac sin B

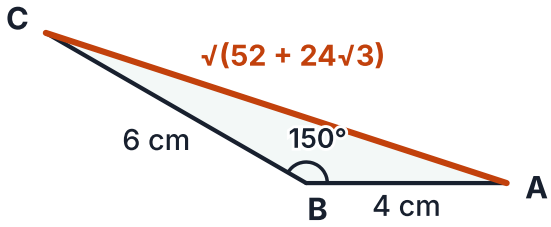
$\sin 150° = \sin 30° = ½$, so area $= 12 \times ½ = 6$ cm²

exact: $\sin 150° = ½$

(b) $AC^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \times \cos 150°$ — cosine rule

$\cos 150° = -\frac{\sqrt{3}}{2}$: $AC^2 = 52 - 48 \times (-\frac{\sqrt{3}}{2}) = 52 + 24\sqrt{3}$

exact: $\cos 150° = -½\sqrt{3}$



area 6 cm²

area 6 cm², $AC^2 = 52 + 24\sqrt{3}$

15 Find the exact value of $\tan 30° + \tan 60°$. Write it as a single fraction with a rational denominator.

$\tan 30° + \tan 60° = \frac{1}{\sqrt{3}} + \sqrt{3}$ — exact values

$= \frac{1}{\sqrt{3}} + \frac{3}{\sqrt{3}} = \frac{4}{\sqrt{3}}$ — common denominator

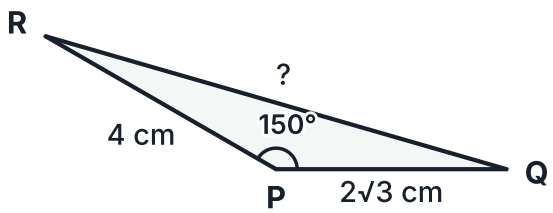
$= \frac{4\sqrt{3}}{3}$ — rationalise: × √3 top and bottom

E · Exam-style question Use the mark scheme to self-assess.

16 (a) Show that $\cos 150° = -\frac{\sqrt{3}}{2}$. [2 marks]

(b) Triangle PQR has $PQ = 2\sqrt{3}$ cm, $PR = 4$ cm, angle $QPR = 150°$. Find the exact length QR. [3 marks]

(c) Find its exact area. [2 marks]



draw a diagram first

(a) $150° = 180° - 30°$ — write θ = 180° − α

As far from the axis as at 30°, other side: $\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$ **M1 A1** — graph: same distance, other side

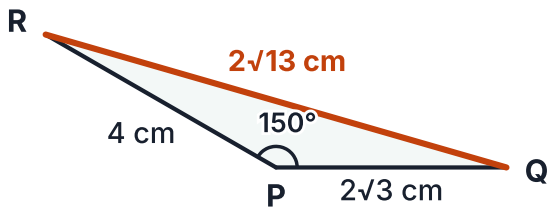
(b) $QR^2 = (2\sqrt{3})^2 + 4^2 - 2 \times 2\sqrt{3} \times 4 \times \cos 150°$ **M1** — cosine rule

$= 12 + 16 - 16\sqrt{3} \times (-\frac{\sqrt{3}}{2}) = 28 + 24 = 52$ **A1** — −16√3 × (−½√3) = +24

$QR = \sqrt{52} = 2\sqrt{13}$ cm **A1** — simplify the surd

(c) Area = ½ × $2\sqrt{3}$ × 4 × sin 150° **M1** — ½ × PQ × PR × sin P

sin 150° = ½: area = $4\sqrt{3} \times \frac{1}{2} = 2\sqrt{3}$ cm² **A1** — exact: sin 150° = ½



QR = 2√13 cm, area 2√3 cm²

area $2\sqrt{3}$ cm²