Trigonometric Identities

Specification: AQA 7357: E5 (proofs E8, A-level) · Edexcel 9MA0: 5.5

INDEPENDENT **You do**

A · Simplifying No calculator.

1 Simplify
(a) $3 - 3\sin^2\theta$
(b) $2\sin^2 4\theta + 2\cos^2 4\theta$
(c) $\cos^2\theta - 1$
(d) $\dfrac{\sin\theta}{\tan\theta\cos\theta}$

2 Simplify
(a) $\dfrac{1-\sin^2\theta}{\cos\theta}$
(b) $(\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta$
(c) $\sin^4\theta + \sin^2\theta\cos^2\theta$
(d) $7 - 7\cos^2(\tfrac{1}{2}\theta)$

3 Write each expression in terms of $\sin\theta$ only.
(a) $3\cos^2\theta - 1$
(b) $\dfrac{\cos^2\theta}{1-\sin\theta}$
(c) $\tan^2\theta\cos^2\theta$

4 (a) Given that $3\sin\theta = 7\cos\theta$, find $\tan\theta$.
(b) Given that $2\sin\theta + 5\cos\theta = 0$, find $\tan\theta$.
(c) Given that $\sin x\cos y = 4\cos x\sin y$ ($\cos x \neq 0$, $\cos y \neq 0$), write $\tan x$ in terms of $\tan y$.

B · One ratio → the others No calculator. Exact answers.

5 $\tan\theta = \dfrac{8}{15}$ and $180° < \theta < 270°$. Find exactly
(a) $\sin\theta$
(b) $\cos\theta$

6 $\sin\theta = -\dfrac{20}{29}$ and $270° < \theta < 360°$. Find exactly
(a) $\cos\theta$
(b) $\tan\theta$

7 $\cos\theta = \dfrac{1}{4}$ and θ is reflex. Find exactly
(a) $\sin\theta$
(b) $\tan\theta$

8 $\tan\theta = -2$ and θ is obtuse. Find exactly
(a) $\sin\theta$
(b) $\cos\theta$

9 $\sin\theta = \dfrac{3}{8}$ and θ is obtuse. Using $\sin^2\theta + \cos^2\theta \equiv 1$ (no triangle), find exactly
(a) $\cos\theta$
(b) $\tan\theta$

C · Reasoning No calculator for this section.

10 $\sin\theta = 2\cos\theta$.
(a) Explain why $\cos\theta$ cannot be 0.
(b) Hence find $\tan\theta$.

11 True or false? Give a reason or a counter-example.
(a) $\sin\theta + \cos\theta \equiv 1$
(b) $1 - \sin^2\theta \equiv \cos^2\theta$
(c) $\sqrt{1-\cos^2\theta} \equiv \sin\theta$

12 Given that $\sin\theta = \dfrac{5}{13}$ and $0° \leq \theta < 360°$:
(a) find the two possible values of $\cos\theta$
(b) state the quadrant of θ for each value.

D · Proofs and eliminating θ No calculator. Full proofs: A-level for AQA (E8), AS for Edexcel (5.5).

13 Prove that
$(1-\cos\theta)(1+\cos\theta) \equiv \tan^2\theta\cos^2\theta$.

14 Prove that
$\dfrac{1}{\cos\theta} - \sin\theta\tan\theta \equiv \cos\theta$.

15 Eliminate θ to find an equation connecting x and y.
(a) $x = 2\sin\theta$, $y = 5\cos\theta$
(b) $x = 3 + \cos\theta$, $y = \sin\theta - 1$

E · Exam-style question No calculator. Use the mark scheme to self-assess.

16 Triangle ABC has $AB = 6$ cm, $BC = 5$ cm and $AC = 4$ cm.
(a) Show that $\cos B = \frac{3}{4}$. [3 marks]
(b) Hence find the exact value of $\sin B$. [2 marks]
(c) Find the exact area of the triangle. [2 marks]