Simple Trigonometric Equations

Specification: AQA 7357: E7 · Edexcel 9MA0: 5.7

INDEPENDENT ## You do

A · Principal values and when solutions exist No calculator.

1 Write down the principal value of
(a) $\sin^{-1}(\frac{1}{2})$
(b) $\cos^{-1}(-\frac{1}{2})$
(c) $\tan^{-1}(-1)$
(d) $\sin^{-1}(-\frac{\sqrt{3}}{2})$

(a) $\sin 30° = \frac{1}{2}$ and 30° is in −90° to 90°: 30°
$\sin^{-1}$: −90° to 90°

(b) $\cos 60° = \frac{1}{2}$; $\cos^{-1}$ gives 0° to 180°, and cos is negative in 90°–180°: 180° − 60° = 120°
$\cos^{-1}$: 0° to 180°

(c) $\tan 45° = 1$; $\tan^{-1}$ gives between −90° and 90° (never ±90°): −45°
$\tan^{-1}$: −90° < PV < 90°

(d) $\sin 60° = \frac{\sqrt{3}}{2}$; in −90° to 90°: −60°
$\sin^{-1}$: −90° to 90°

2 Which of these equations have solutions? Explain.
(a) $\sin\theta = 0.95$
(b) $\cos\theta = -1.2$
(c) $\tan\theta = 40$
(d) $3\sin\theta = 4$
(e) $2\cos\theta + 3 = 0$

(a) Yes: 0.95 is between −1 and 1. −1 ≤ sin θ ≤ 1

(b) No: −1.2 is less than −1. −1 ≤ cos θ ≤ 1

(c) Yes: tan takes every value. tan: every value

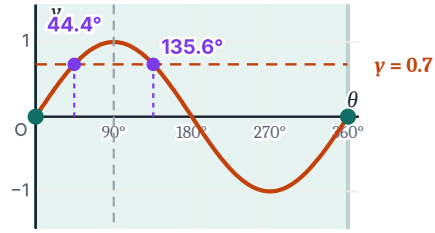
(d) $\sin\theta = \frac{4}{3}$, more than 1: no. rearrange, then check

(e) $\cos\theta = -1.5$, less than −1: no. rearrange, then check

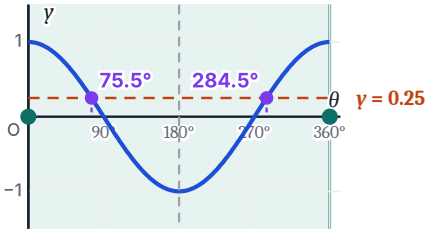
B · Solving in 0° ≤ θ ≤ 360° Calculator, except question 6.

3 Solve each equation for $0° \le \theta \le 360°$. Give angles to 1 d.p. unless exact.
(a) $\sin\theta = 0.7$
(b) $\cos\theta = 0.25$

(a) sin θ = 0.7



(b) cos θ = 0.25



axes; shade the intervals

(a) Two crossings in the interval. PV $\sin^{-1}(0.7) = 44.4°$ sketch; PV
$\theta = 180° - 44.4° = 135.6°$ 180° − PV
Check: all in the interval; $\theta = 44.4°, 135.6°$ (2) check the interval

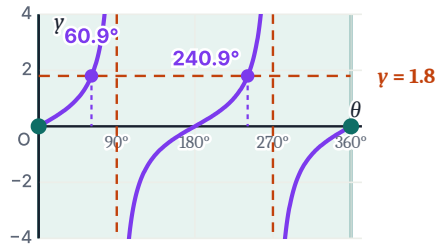
(b) Two crossings in the interval. PV $\cos^{-1}(0.25) = 75.5°$ sketch; PV
$\theta = 360° - 75.5° = 284.5°$ 360° − PV
Check: all in the interval; $\theta = 75.5°, 284.5°$ (2) check the interval

4 Solve each equation for $0° \le \theta \le 360°$. Give angles to 1 d.p. unless exact.

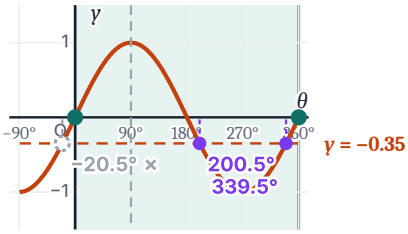
(a) $\tan\theta = 1.8$

(b) $\sin\theta = -0.35$

(a) tan θ = 1.8



(b) sin θ = −0.35



axes; shade the intervals

(a) Two crossings in the interval. PV $\tan^{-1}(1.8) = 60.9°$ — sketch; PV

$\theta = 60.9° + 180° = 240.9°$ — PV + 180°

Check: all in the interval; $\theta = 60.9°, 240.9°$ (2) — check the interval

(b) Two crossings in the interval. PV $\sin^{-1}(-0.35) = -20.5°$ — sketch; PV

$\theta = 180° - (-20.5°) = 200.5°$; $-20.5° + 360° = 339.5°$ — 180° − PV; ± 360°

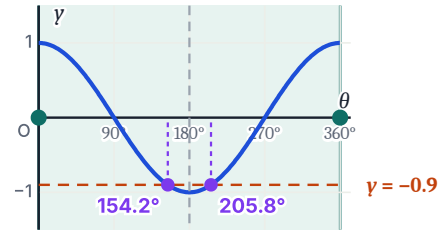
Check: ~~$-20.5°$~~ × (below 0°); $\theta = 200.5°, 339.5°$ (2) — check the interval

5 Solve each equation for $0° \le \theta \le 360°$. Give angles to 1 d.p. unless exact.

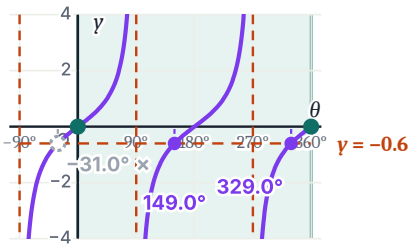
(a) $\cos\theta = -0.9$

(b) $\tan\theta = -0.6$

(a) cos θ = −0.9



(b) tan θ = −0.6



axes; shade the intervals

(a) Two crossings in the interval. PV $\cos^{-1}(-0.9) = 154.2°$ — sketch; PV

$\theta = 360° - 154.2° = 205.8°$ — 360° − PV

Check: all in the interval; $\theta = 154.2°, 205.8°$ (2) — check the interval

(b) Two crossings in the interval. PV $\tan^{-1}(-0.6) = -31.0°$ — sketch; PV

$\theta = (-31.0°) + 180° = 149.0°$; $149.0° + 180° = 329.0°$ — PV + 180°; ± 180°

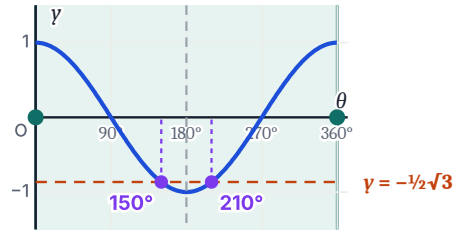
Check: ~~$-31.0°$~~ × (below 0°); $\theta = 149.0°, 329.0°$ (2) — check the interval

6 Without a calculator, solve each equation for $0° \le \theta \le 360°$. Give exact angles.

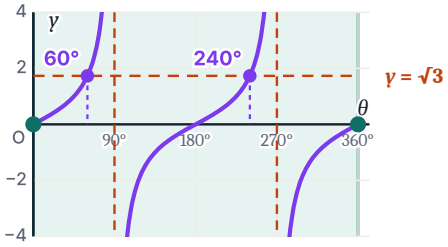
(a) $\cos\theta = -\frac{\sqrt{3}}{2}$

(b) $\tan\theta = \sqrt{3}$

(a) cos θ = −½√3



(b) tan θ = √3



axes; shade the intervals

(a) Two crossings in the interval. PV $\cos^{-1}(-\frac{\sqrt{3}}{2}) = 150°$ — sketch; PV

$\theta = 360° - 150° = 210°$ — 360° − PV

Check: all in the interval; $\theta = 150°, 210°$ (2) — check the interval

(b) Two crossings in the interval. PV $\tan^{-1}(\sqrt{3}) = 60°$ — sketch; PV

$\theta = 60° + 180° = 240°$ — PV + 180°

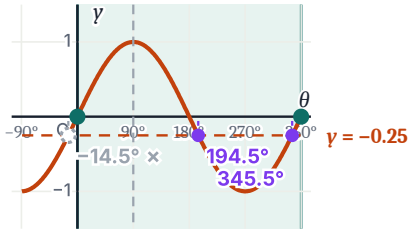
Check: all in the interval; $\theta = 60°, 240°$ (2) — check the interval

7 Solve each equation for $0° \le \theta \le 360°$. Give angles to 1 d.p. unless exact.

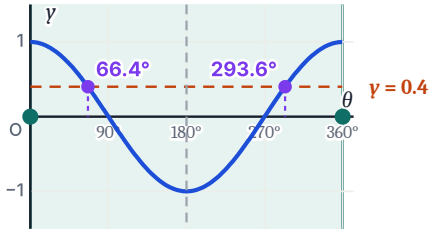
(a) $4\sin\theta + 1 = 0$

(b) $5\cos\theta = 2$

(a) 4 sin θ + 1 = 0



(b) 5 cos θ = 2



axes; shade the intervals

(a) $4\sin\theta = -1$, so $\sin\theta = -0.25$ — − 1, then ÷ 4

Two crossings in the interval. PV $\sin^{-1}(-0.25) = -14.5°$ — sketch; PV

$\theta = 180° - (-14.5°) = 194.5°$; $-14.5° + 360° = 345.5°$ — 180° − PV; ± 360°

Check: ~~−14.5°~~ × (below 0°); $\theta = 194.5°, 345.5°$ (2) — check the interval

(b) ÷ 5: $\cos\theta = 0.4$ — ÷ 5: cos θ = ...

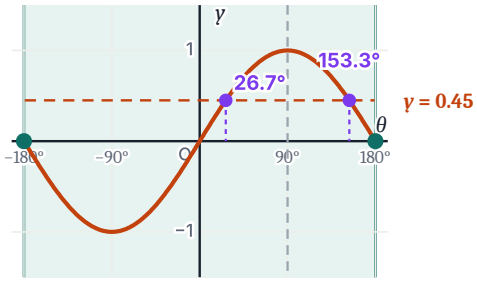
Two crossings in the interval. PV $\cos^{-1}(0.4) = 66.4°$ — sketch; PV

$\theta = 360° - 66.4° = 293.6°$ — 360° − PV

Check: all in the interval; $\theta = 66.4°, 293.6°$ (2) — check the interval

C · Other intervals and reasoning Calculator for 8 and 9. No calculator for 10 and 11.

8 Solve $\sin\theta = 0.45$ for $-180° \le \theta \le 180°$. Give angles to 1 d.p. unless exact.



axes; shade the interval

Two crossings in the interval. PV $\sin^{-1}(0.45) = 26.7°$

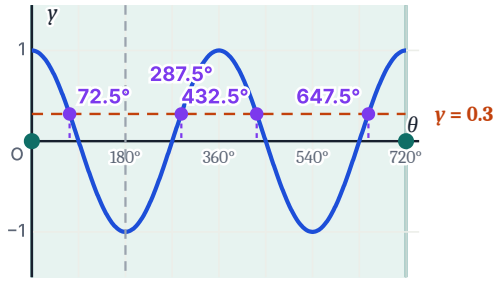
sketch; PV

$\theta = 180° - 26.7° = 153.3°$ 180° − PV

Check: all in the interval; $\theta = 26.7°, 153.3°$ (2)

check the interval

9 Solve $\cos\theta = 0.3$ for $0° \le \theta \le 720°$. Give angles to 1 d.p. unless exact.



axes; shade the interval

Four crossings in the interval. PV $\cos^{-1}(0.3) = 72.5°$

sketch; PV

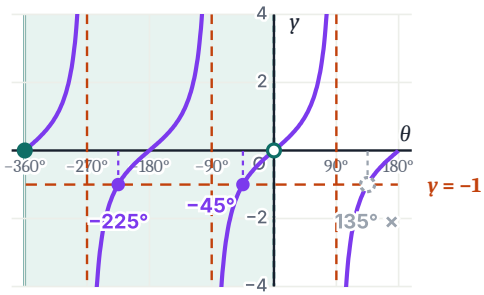
$\theta = 360° - 72.5° = 287.5°$; $287.5° + 360° = 647.5°$; $72.5° + 360° = 432.5°$

360° − PV; ± 360°

Check: all in the interval; $\theta = 72.5°, 287.5°, 432.5°, 647.5°$ (4)

check the interval

10 Without a calculator, solve $\tan\theta = -1$ for $-360° \le \theta < 0°$. Give exact angles.



axes; shade the interval

Two crossings in the interval. PV $\tan^{-1}(-1) = -45°$

sketch; PV

$\theta = (-45°) + 180° = 135°$; $-45° - 180° = -225°$

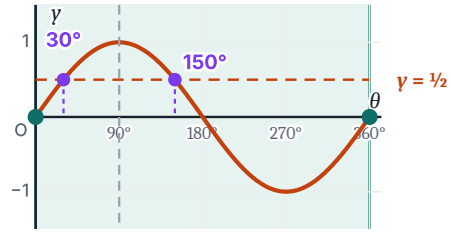
PV + 180°; ± 180°

Check: ~~135°~~ × (above 0°); $\theta = -225°, -45°$ (2)

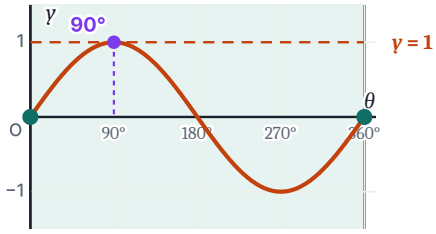
check the interval

11 How many solutions does each equation have in $0° \le \theta \le 360°$? No calculator: explain with the graph.
(a) $\sin\theta = ½$
(b) $\sin\theta = 1$
(c) $\sin\theta = 0$

(a) sin θ = ½

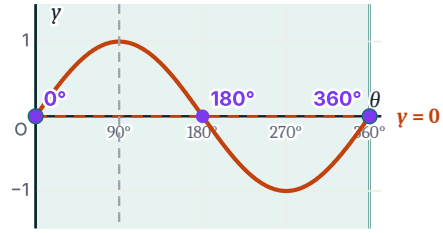


(b) sin θ = 1



axes; shade the intervals

(c) sin θ = 0



(a) $y = ½$ crosses the curve twice: $30°, 150°$: **2** solutions. — count the crossings

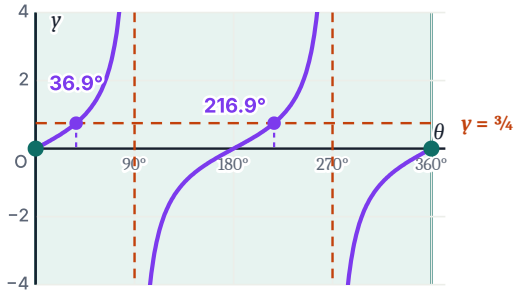
(b) $y = 1$ touches the curve once, at the maximum $90°$: **1** solution. — touches the maximum

(c) $y = 0$ is the axis: the zeros $0°, 180°, 360°$ (both ends included): **3** solutions. — the zeros; both ends

D · a sin θ = b cos θ

Show that cos θ ≠ 0 before you divide. Calculator for 12 and 13; no calculator for 14.

12 Solve $4\sin\theta = 3\cos\theta$ for $0° \le \theta \le 360°$. Give angles to 1 d.p. unless exact.



axes; shade the interval

If $\cos\theta = 0$, $\sin\theta = \pm1$ and the equation says $\pm4 = 0$: impossible. ÷ $4\cos\theta$: $\tan\theta = ¾$

÷ cos θ (cos θ ≠ 0)

Two crossings in the interval. PV $\tan^{-1}(¾) = 36.9°$

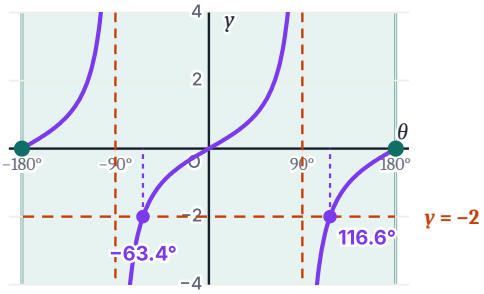
sketch; PV

$\theta = 36.9° + 180° = 216.9°$ — PV + 180°

Check: all in the interval; $\theta = 36.9°, 216.9°$ (2)

check the interval

13 Solve $\sin\theta + 2\cos\theta = 0$ for $-180° \le \theta \le 180°$. Give angles to 1 d.p. unless exact.



axes; shade the interval

If $\cos\theta = 0$, $\sin\theta = \pm1$ and the equation says $\pm1 = 0$: impossible. ÷ $\cos\theta$: $\tan\theta + 2 = 0$

÷ cos θ (cos θ ≠ 0)

$\tan\theta = -2$ — rearrange: tan θ = ...

Two crossings in the interval. PV $\tan^{-1}(-2) = -63.4°$

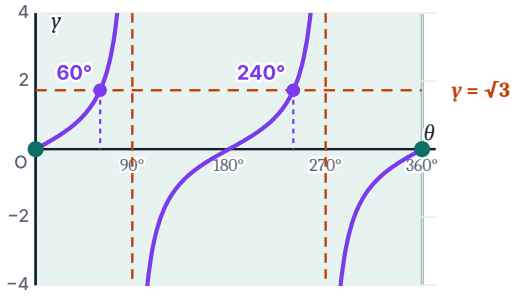
sketch; PV

$\theta = (-63.4°) + 180° = 116.6°$ — PV + 180°

Check: all in the interval; $\theta = -63.4°, 116.6°$ (2)

check the interval

14 Without a calculator, solve $\sqrt{2}\sin\theta = \sqrt{6}\cos\theta$ for $0° \le \theta \le 360°$. Give exact angles.



axes; shade the interval

If $\cos\theta = 0$, $\sin\theta = \pm 1$ and the equation says $\pm\sqrt{2} = 0$: impossible. $\div \sqrt{2}\cos\theta$: $\tan\theta = \sqrt{6} \div \sqrt{2} = \sqrt{3}$

$\div \cos\theta$ ($\cos\theta \neq 0$)

Two crossings in the interval. PV $\tan^{-1}(\sqrt{3}) = 60°$

sketch; PV

$\theta = 60° + 180° = 240°$ PV + 180°

Check: all in the interval; $\theta = 60°, 240°$ (2) check the interval

E · Exam-style question Calculator. 6 marks.

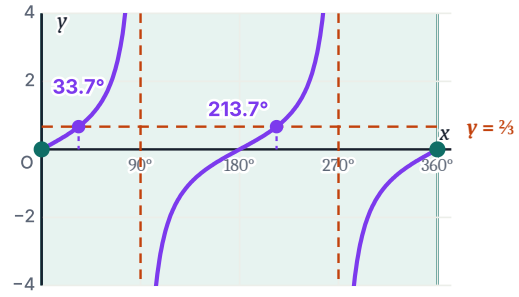
15 *Exam-style.*

(a) Show that $3\sin x = 2\cos x$ can be written as $\tan x = \frac{2}{3}$, explaining why you may divide by $\cos x$. [2 marks]

(b) Hence solve $3\sin x = 2\cos x$ for $0° \le x \le 360°$, giving your answers to 1 d.p. [3 marks]

(c) How many solutions does $3\sin x = 2\cos x$ have for $0° \le x \le 1080°$? [1 mark]

tan x = ⅔



axes; shade the intervals

(a) If $\cos x = 0$, then $\sin x = \pm 1$ and the equation says $\pm 3 = 0$: impossible. So $\cos x \neq 0$. **B1** — check cos x ≠ 0

$\div 3\cos x$: $\tan x = \frac{2}{3}$ **B1** — ÷ cos x (cos x ≠ 0)

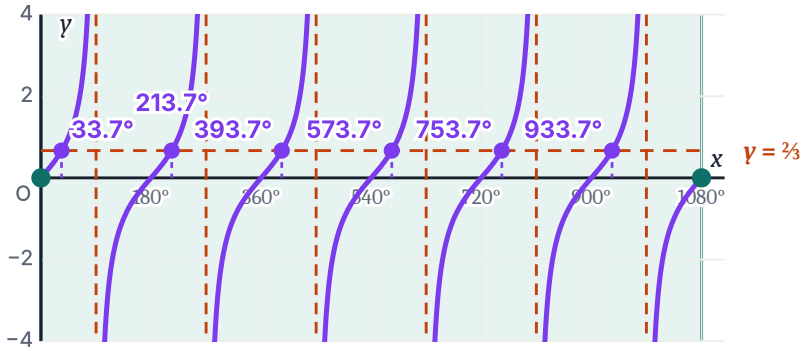
(b) Two crossings in the interval. PV $\tan^{-1}(\frac{2}{3}) = 33.7°$ **M1 A1** — sketch; PV

$x = 33.7° + 180° = 213.7°$ **A1** — PV + 180°

Check: all in the interval; $x = 33.7°, 213.7°$ (2) — check the interval

(c) tan repeats every 180°: two solutions in every 360°, so $3 \times 2 = 6$ in 0°–1080° ($33.7° + 180°n$ for n = 0 to 5). **B1**

period 180°: 2 per 360°



(c) two per 180°: 6 solutions

(c) 6 solutions in 0° ≤ x ≤ 1080°