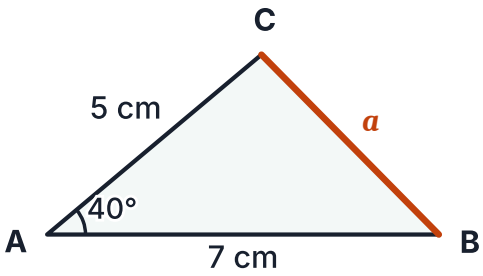
The Cosine Rule

Specification: AQA 7357: E1 · Edexcel 9MA0: 5.1

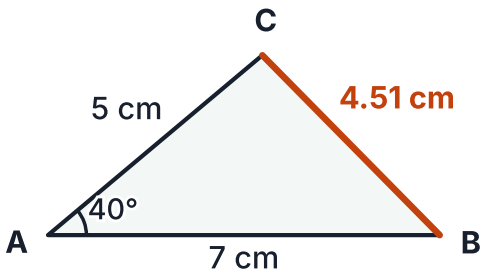
INDEPENDENT **You do**

A · Find the missing side

1 Find a.

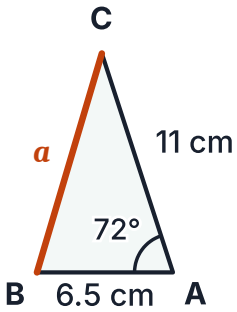


$a^2 = 5^2 + 7^2 - 2(5)(7)\cos 40°$ — cosine rule: substitute
$a^2 = 74 - 70\cos 40° = 20.376...$ — calculator: full value
$a = 4.5140...$, so $a = 4.51$ cm — square root, 3 s.f.

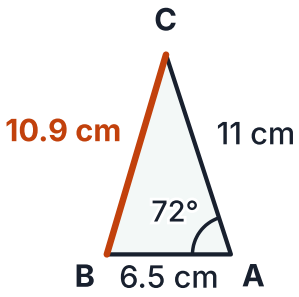


a = 4.51 cm

2 Find a.

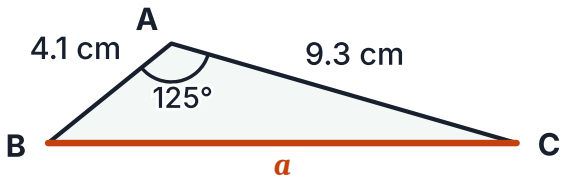


$a^2 = 11^2 + 6.5^2 - 2(11)(6.5)\cos 72°$ — cosine rule: substitute
$a^2 = 163.25 - 143\cos 72° = 119.060...$ — calculator: full value
$a = 10.911...$, so $a = 10.9$ cm — square root, 3 s.f.

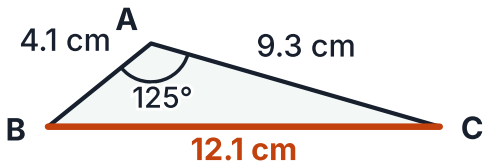


a = 10.9 cm

3 Find a.

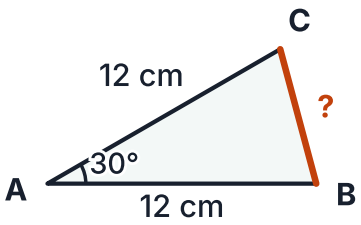


$a^2 = 9.3^2 + 4.1^2 - 2(9.3)(4.1)\cos 125°$ — cosine rule: substitute
$a^2 = 103.3 + 43.740... = 147.04...$ — $\cos 125° < 0$, so we add
$a = 12.126...$, so $a = 12.1$ cm — square root, 3 s.f.



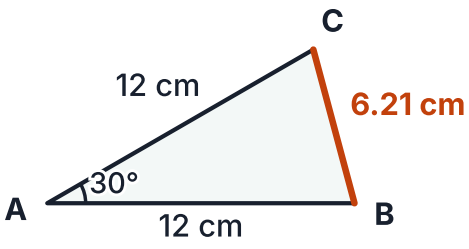
a = 12.1 cm

4 An isosceles triangle has two sides of 12 cm with 30° between them. Find the length of the third side.



draw a diagram first

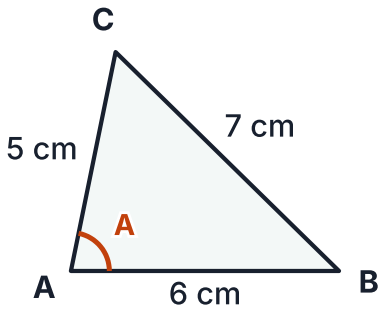
$a^2 = 12^2 + 12^2 - 2(12)(12)\cos 30°$ — cosine rule: substitute
$a^2 = 288 - 288\cos 30° = 38.584...$ — calculator: full value
$a = 6.2116...$, so $a = 6.21$ cm — square root, 3 s.f.



third side 6.21 cm

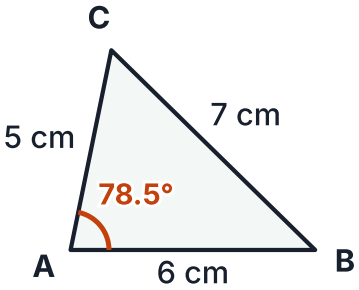
B · Find the missing angle

5 Find angle A.



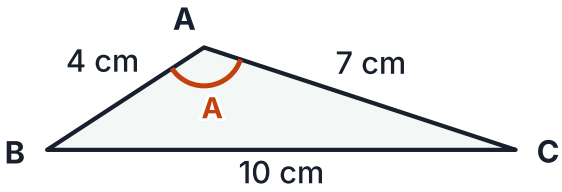
$\cos A = \dfrac{5^2 + 6^2 - 7^2}{2(5)(6)} = \dfrac{12}{60} = 0.2$ — rearranged cosine rule

$A = 78.463...°$, so $A = 78.5°$ — $\cos^{-1}$, 1 d.p.



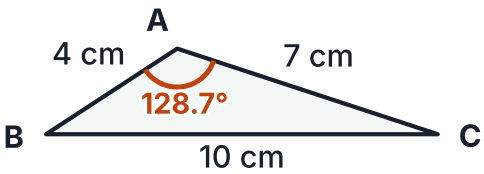
A = 78.5°

6 Find angle A.



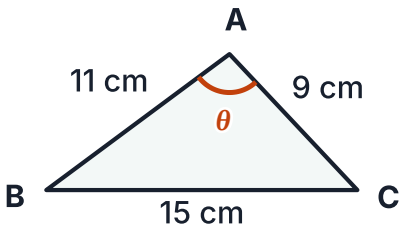
$\cos A = \dfrac{7^2 + 4^2 - 10^2}{2(7)(4)} = \dfrac{-35}{56}$ — negative ⇒ obtuse

$A = 128.68...°$, so $A = 128.7°$ — $\cos^{-1}$, 1 d.p.



A = 128.7° (obtuse)

7 A triangle has sides 9 cm, 11 cm and 15 cm. Find the largest angle.



draw a diagram first

The largest angle is opposite the 15 cm side.

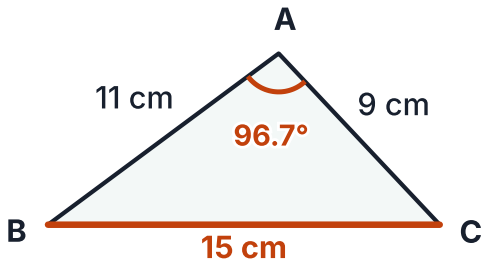
largest angle: opposite longest side

$\cos\theta = \dfrac{9^2 + 11^2 - 15^2}{2(9)(11)} = \dfrac{-23}{198}$

rearranged cosine rule

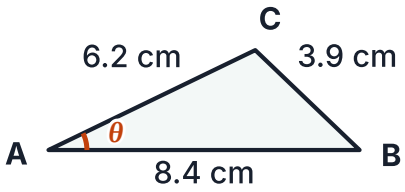
$\theta = 96.670...°$, so $\theta = 96.7°$

$\cos^{-1}$, 1 d.p.



largest angle 96.7°, opposite 15 cm

8 A triangle has sides 3.9 cm, 6.2 cm and 8.4 cm. Find the smallest angle.



draw a diagram first

The smallest angle is opposite the 3.9 cm side.

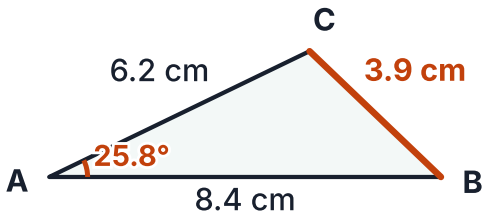
smallest angle: opposite shortest side

$\cos\theta = \dfrac{6.2^2 + 8.4^2 - 3.9^2}{2(6.2)(8.4)} = 0.9004...$

rearranged cosine rule

$\theta = 25.783...°$, so $\theta = 25.8°$

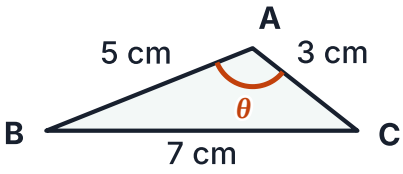
$\cos^{-1}$, 1 d.p.



smallest angle 25.8°, opposite 3.9 cm

C · Reasoning No calculator for this section.

9 A triangle has sides 3 cm, 5 cm and 7 cm. Show that its largest angle is exactly 120°.



draw a diagram first

The largest angle is opposite the 7 cm side.

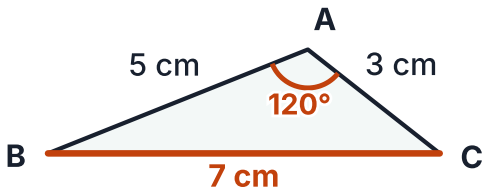
largest angle: opposite longest side

$\cos\theta = \dfrac{3^2 + 5^2 - 7^2}{2(3)(5)} = \dfrac{-15}{30} = -\dfrac{1}{2}$

rearranged cosine rule

$\theta = 120°$

$\cos 120° = -½$



largest angle exactly 120°

10 Use the cosine rule to explain why no triangle has sides 4 cm, 6 cm and 11 cm.

$\cos\theta = \dfrac{4^2 + 6^2 - 11^2}{2(4)(6)} = \dfrac{-69}{48} = -1.4375$

rearranged cosine rule

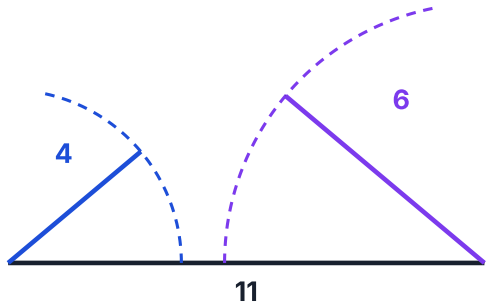
But $-1 \le \cos\theta \le 1$ for every angle, so no such angle exists.

$-1 \le \cos\theta \le 1$

Also $4 + 6 < 11$: the two short sides cannot meet.

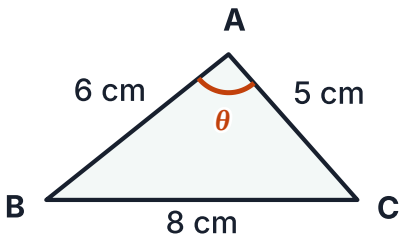
triangle inequality

4 + 6 < 11: the sides cannot meet



4 + 6 < 11: no triangle

11 Is a triangle with sides 5 cm, 6 cm and 8 cm acute, right-angled or obtuse? Explain without finding the angle.

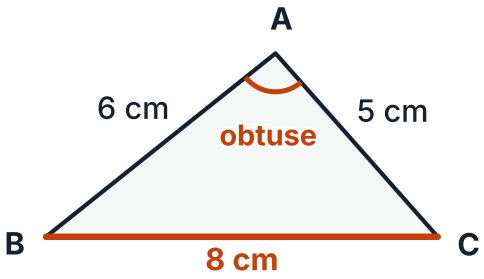


draw a diagram first

$5^2 + 6^2 = 61$ and $8^2 = 64$ — compare a^2 with $b^2 + c^2$

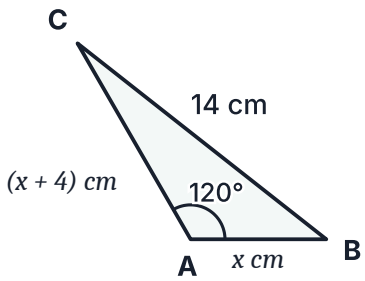
$a^2 > b^2 + c^2$, so $\cos\theta < 0$ — a^2 bigger ⇒ $\cos\theta < 0$

The largest angle is obtuse. — negative ⇒ obtuse



$8^2 > 5^2 + 6^2$: the angle opposite 8 cm is obtuse (92.9°)

12 $AB = x$ cm, $AC = (x + 4)$ cm, angle $BAC = 120°$ and $BC = 14$ cm. Find x.



draw a diagram first

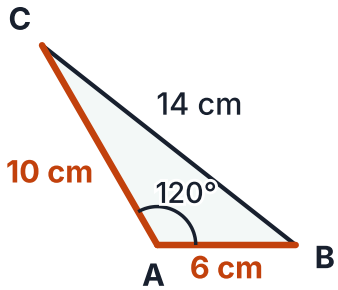
$14^2 = x^2 + (x + 4)^2 - 2x(x + 4)\cos 120°$ — 14 is opposite 120°

$196 = x^2 + x^2 + 8x + 16 + x^2 + 4x$ — cos 120° = −½; expand

$3x^2 + 12x - 180 = 0 \Rightarrow x^2 + 4x - 60 = 0$ — collect terms; ÷ 3

$(x + 10)(x - 6) = 0$ — factorise

$x = 6$ — reject: x > 0

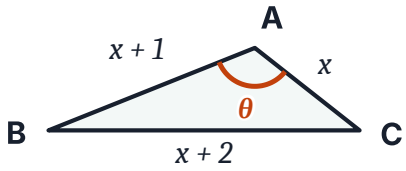


x = 6: AB = 6 cm, AC = 10 cm

13 A triangle has sides x cm, $(x + 1)$ cm and $(x + 2)$ cm. The largest angle is θ.

(a) Show that $\cos\theta = \dfrac{x - 3}{2x}$.

(b) Given that θ = 120°, find x.



draw a diagram first

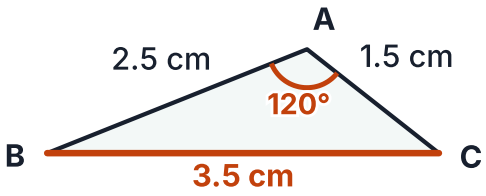
(a) $\cos\theta = \dfrac{x^2 + (x + 1)^2 - (x + 2)^2}{2x(x + 1)}$ — θ opposite x + 2

$= \dfrac{x^2 - 2x - 3}{2x(x + 1)} = \dfrac{(x - 3)(x + 1)}{2x(x + 1)} = \dfrac{x - 3}{2x}$

expand; factorise; cancel (x + 1)

(b) $\dfrac{x - 3}{2x} = -\dfrac{1}{2} \Rightarrow 2x - 6 = -2x$ — cos 120° = −½; × 2x

$4x = 6 \Rightarrow x = 1.5$ — solve



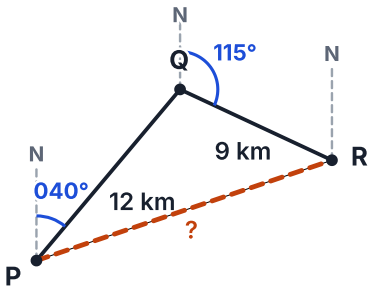
(b) x = 1.5: sides 1.5, 2.5 and 3.5 cm

E · Exam-style question Use the mark scheme to self-assess.

14 A ship sails 12 km from port P on a bearing of 040° to a point Q. It then sails 9 km on a bearing of 115° to a point R.

(a) Find the distance PR. [3 marks]

(b) Find the bearing of R from P. [3 marks]



draw a diagram: North lines first

(a) $\angle PQR = 220° - 115° = 105°$ **B1** — back bearing: + 180°

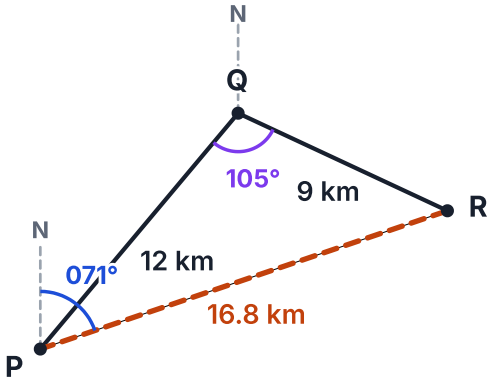
$PR^2 = 12^2 + 9^2 - 2(12)(9)\cos 105°$ **M1** — cosine rule: substitute

$PR^2 = 280.90\ldots \Rightarrow PR = 16.760\ldots$, so 16.8 km **A1** — square root, 3 s.f.

(b) $\cos \angle QPR = \dfrac{12^2 + 16.760\ldots^2 - 9^2}{2(12)(16.760\ldots)}$ **M1** — rearranged cosine rule

$\angle QPR = 31.244\ldots°$ **A1** — $\cos^{-1}$

Bearing = 040° + 31.2° = 071.2°, so **071°** **A1** — add to the bearing of Q



PR = 16.8 km; bearing of R from P is 071°