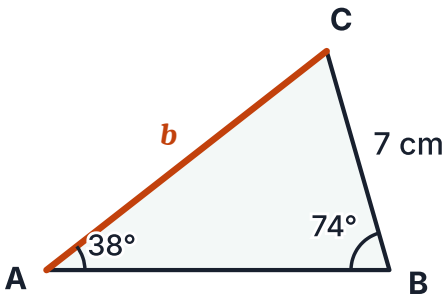
The Sine Rule

Specification: AQA 7357: E1 · Edexcel 9MA0: 5.1

INDEPENDENT **You do**

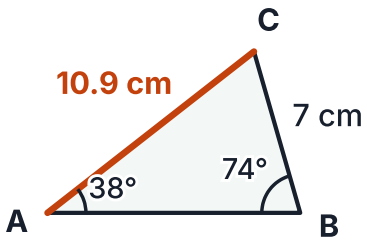
A · Find the missing side

1 Find b.



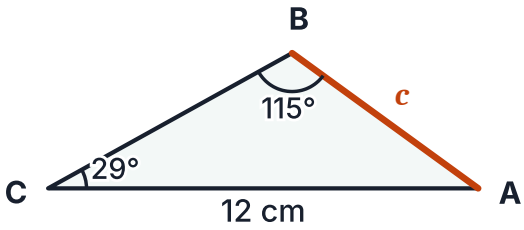
$\frac{b}{\sin 74°} = \frac{7}{\sin 38°}$ — pair: a = 7 with 38°

$b = \frac{7 \sin 74°}{\sin 38°} = 10.929\ldots$, so $b = 10.9$ cm — × sin 74°; 3 s.f.



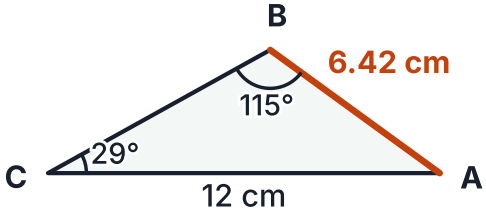
b = 10.9 cm

2 Find c.



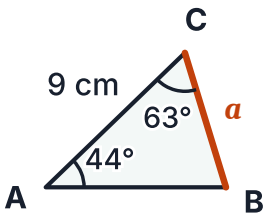
$\frac{c}{\sin 29°} = \frac{12}{\sin 115°}$ — pair: b = 12 with 115°

$c = \frac{12 \sin 29°}{\sin 115°} = 6.4191\ldots$, so $c = 6.42$ cm — × sin 29°; 3 s.f.



c = 6.42 cm

3 In triangle ABC, $A = 44°$, $C = 63°$ and $b = 9$ cm. Find a.

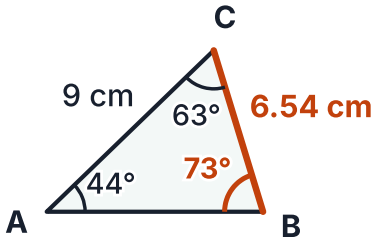


draw a diagram first

$B = 180° - 44° - 63° = 73°$ — third angle first

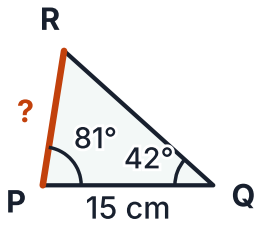
$\frac{a}{\sin 44°} = \frac{9}{\sin 73°}$ — pair: b = 9 with 73°

$a = 6.5375...$, so $a = 6.54$ cm — × sin 44°; 3 s.f.



B = 73°, a = 6.54 cm

4 In triangle PQR, angle $P = 81°$, angle $Q = 42°$ and $PQ = 15$ cm. Find PR.

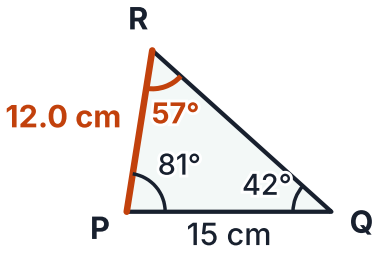


draw a diagram first

$R = 180° - 81° - 42° = 57°$ — third angle first

$\frac{PR}{\sin 42°} = \frac{15}{\sin 57°}$ — PR is opposite Q; PQ opposite R

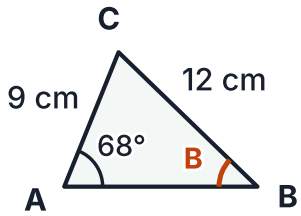
$PR = 11.967...$, so $PR = 12.0$ cm — × sin 42°; 3 s.f.



R = 57°, PR = 12.0 cm

B · Find the missing angle

5 In triangle ABC, $a = 12$ cm, $b = 9$ cm and $A = 68°$. Find B.

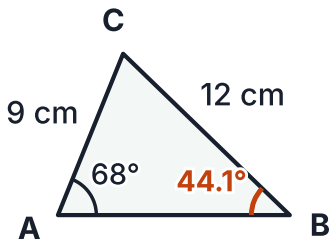


draw a diagram first

$\frac{\sin B}{9} = \frac{\sin 68°}{12}$ — sines on top

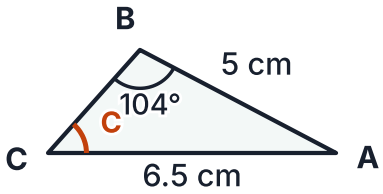
$\sin B = 0.69539...$ — × 9

$B = 44.058...°$, so $B = 44.1°$ — $\sin^{-1}$; 9 < 12 so acute



B = 44.1°

6 In triangle ABC, $b = 6.5$ cm, $c = 5$ cm and $B = 104°$. Find C.

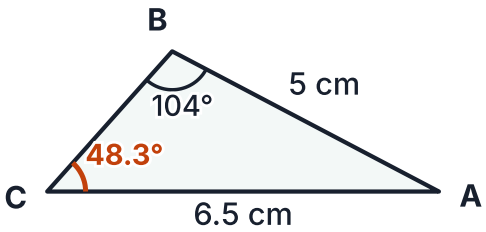


draw a diagram first

$\frac{\sin C}{5} = \frac{\sin 104°}{6.5}$ — sines on top

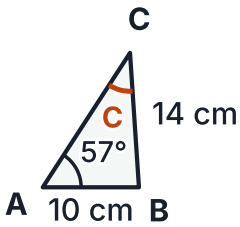
$\sin C = 0.74638...$ — × 5

$C = 48.277...°$, so $C = 48.3°$ — $\sin^{-1}$; B is obtuse, so C is acute



C = 48.3°

7 In triangle ABC, $a = 14$ cm, $c = 10$ cm and $A = 57°$. Find C and B.

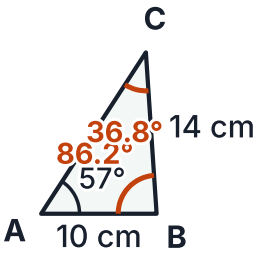


draw a diagram first

$\sin C = \dfrac{10 \sin 57°}{14} = 0.59906\ldots$ — sines on top; × 10

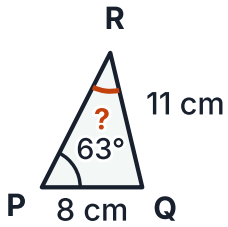
$C = 36.801\ldots°$, so $C = 36.8°$ — $\sin^{-1}$; 10 < 14 so acute

$B = 180° - 57° - 36.8° = 86.2°$ — angles add to 180°



C = 36.8°, B = 86.2°

8 In triangle PQR, $PQ = 8$ cm, $QR = 11$ cm and angle $P = 63°$. Find angle R.

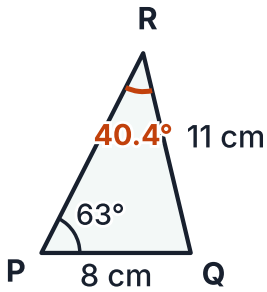


draw a diagram first

Pairs: $QR = 11$ with P; $PQ = 8$ with R. — side ↔ opposite angle

$\sin R = \dfrac{8 \sin 63°}{11} = 0.64800\ldots$ — sines on top

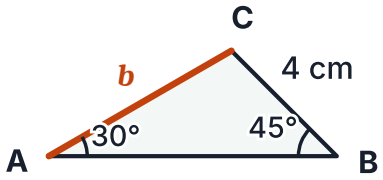
$R = 40.391\ldots°$, so $R = 40.4°$ — $\sin^{-1}$; 8 < 11 so acute



R = 40.4°

C · Reasoning No calculator for this section.

9 In triangle ABC, $A = 30°$, $B = 45°$ and $a = 4$ cm. Show that $b = 4\sqrt{2}$ cm.

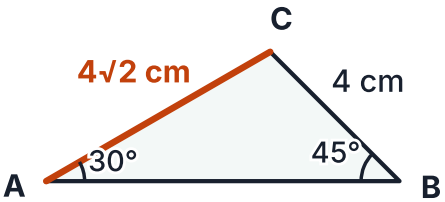


draw a diagram first

$b = \dfrac{4 \sin 45°}{\sin 30°}$ — sine rule: sides on top

$b = \dfrac{4 \times \frac{\sqrt{2}}{2}}{\frac{1}{2}}$ — sin 45° = √⅔, sin 30° = ½

$b = 4\sqrt{2}$ — ÷ ½ is × 2



b = 4√2 ≈ 5.66 cm

10 Which rule would you use first? Do not solve.
(a) $a = 5$, $b = 8$, $C = 70°$: find c.
(b) $A = 40°$, $B = 75°$, $c = 9$: find a.
(c) sides 4, 6, 7: find the largest angle.
(d) $a = 10$, $A = 50°$, $b = 8$: find B.

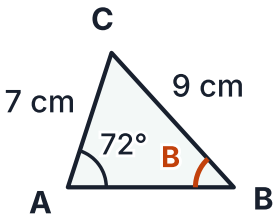
(a) Cosine rule: two sides and the angle between them. — no pair: cosine

(b) Sine rule, after $C = 65°$ (pair c, C). — third angle, then sine

(c) Cosine rule: three sides. — no angle: cosine

(d) Sine rule: pair a, A. — pair known: sine

11 In triangle ABC, $a = 9$ cm, $b = 7$ cm and $A = 72°$. Explain why angle B must be acute.



draw a diagram first

The larger angle is opposite the larger side.

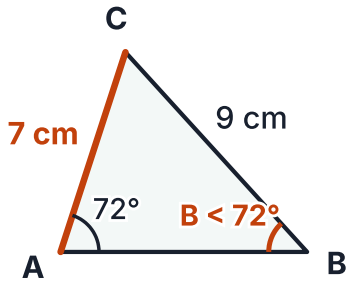
side ↔ opposite angle

$b < a$, so $B < A = 72°$.

compare the sides

So $B < 90°$: acute.

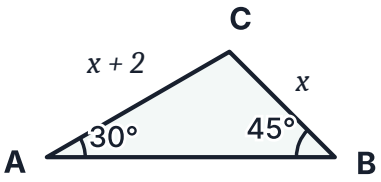
less than 72°



shorter side 7 cm, so the smaller angle B

D · Multi-step and algebra

12 In triangle ABC, angle $A = 30°$, angle $B = 45°$, $BC = x$ cm and $AC = (x + 2)$ cm. Find the exact value of x.



draw a diagram first

$$\frac{x}{\sin 30°} = \frac{x+2}{\sin 45°}$$

BC opposite A; AC opposite B

$$2x = \sqrt{2}(x + 2)$$

sin 30° = ½, sin 45° = √⅔

$$x(2 - \sqrt{2}) = 2\sqrt{2}$$

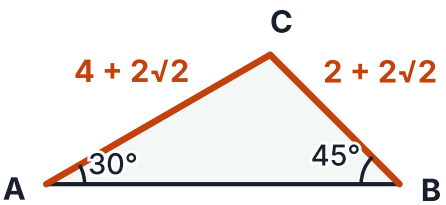
x terms to one side

$$x = \frac{2\sqrt{2}}{2 - \sqrt{2}} = \frac{2\sqrt{2}(2 + \sqrt{2})}{2}$$

rationalise: × (2 + √2)

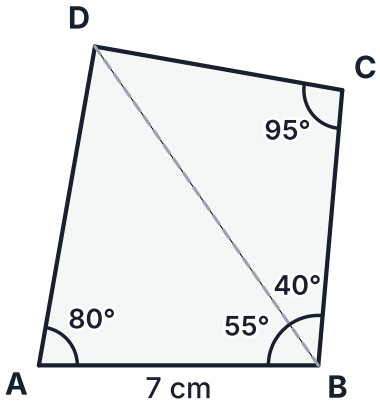
$$x = 2 + 2\sqrt{2}$$

simplify



x = 2 + 2√2 ≈ 4.83

13 In quadrilateral $ABCD$, $AB = 7$ cm, angle $BAD = 80°$, angle $ABD = 55°$, angle $DBC = 40°$ and angle $BCD = 95°$.
(a) Find BD.
(b) Find CD.

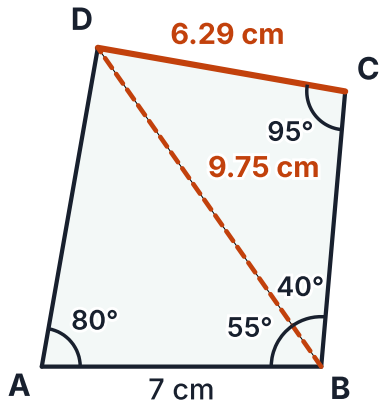


(a) $\angle ADB = 180° - 80° - 55° = 45°$ — third angle in ABD

$\dfrac{BD}{\sin 80°} = \dfrac{7}{\sin 45°} \Rightarrow BD = 9.7491\ldots$, so $BD = 9.75$ cm — sine rule; keep the full value

(b) $\dfrac{CD}{\sin 40°} = \dfrac{9.7491\ldots}{\sin 95°}$ — triangle BCD: pair BD with 95°

$CD = 6.2905\ldots$, so $CD = 6.29$ cm — × sin 40°; 3 s.f.



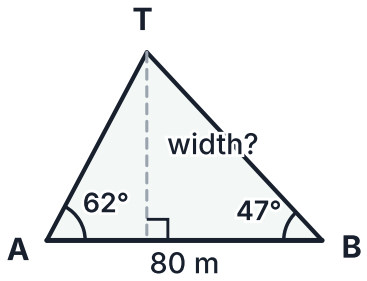
(a) BD = 9.75 cm (b) CD = 6.29 cm

E · Exam-style question Use the mark scheme to self-assess.

14 Points *A* and *B* are 80 m apart on a straight river bank. A tree *T* is on the opposite bank. Angle *TAB* = 62° and angle *TBA* = 47°.

(a) Find the distance *AT*. [3 marks]

(b) Find the width of the river. [2 marks]



draw a diagram first

(a) $\angle ATB = 180° - 62° - 47° = 71°$ **B1**

third angle first

$\frac{AT}{\sin 47°} = \frac{80}{\sin 71°}$ **M1**

AT opposite B; 80 opposite T

$AT = 61.879...$, so 61.9 m **A1**

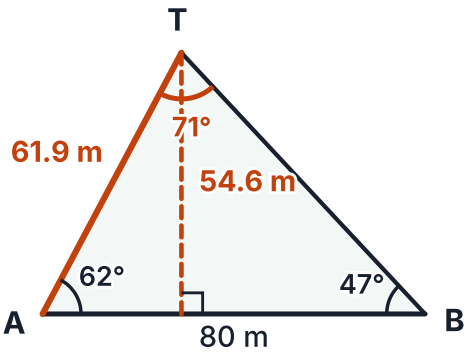
× sin 47°; 3 s.f.

(b) Width = $AT \sin 62°$ **M1**

right-angled: opp = hyp × sin

= 54.636..., so 54.6 m **A1**

3 s.f., with units



(a) AT = 61.9 m (b) width 54.6 m