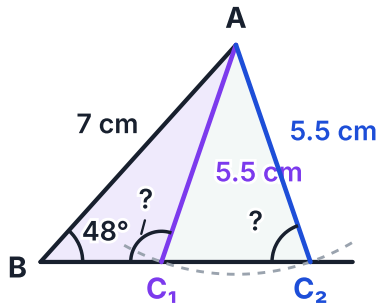
The Ambiguous Case

Specification: AQA 7357: E1 · Edexcel 9MA0: 5.1

INDEPENDENT **You do**

A · Find both possible angles

1 In triangle ABC, AB = 7 cm, AC = 5.5 cm and angle ABC = 48°. Find the two possible values of angle ACB.



draw a diagram: swing AC

$$\frac{\sin C}{7} = \frac{\sin 48°}{5.5}$$ sine rule: sines on top

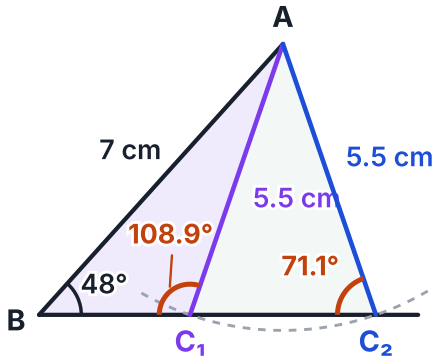
$$\sin C = \frac{7 \sin 48°}{5.5} = 0.94582...$$ × 7

$C = 71.053...°$, so $C = 71.1°$ sin⁻¹: the acute answer

$180° - 71.053...° = 108.94...°$ 180° − θ: the obtuse answer

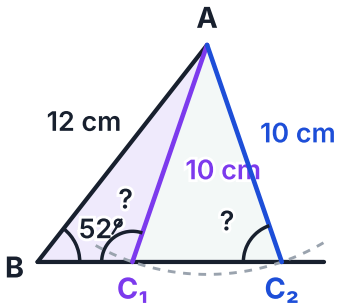
$B + C = 48° + 108.9° = 156.9° < 180°$: it fits, so $C = 108.9°$ too.

180° − θ: does it fit?



C = 71.1° or 108.9°

2 In triangle ABC, AB = 12 cm, AC = 10 cm and angle ABC = 52°. Find the two possible values of angle ACB.



draw a diagram: swing AC

$$\frac{\sin C}{12} = \frac{\sin 52°}{10}$$ sine rule: sines on top

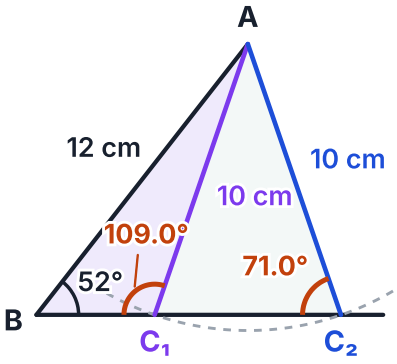
$$\sin C = \frac{12 \sin 52°}{10} = 0.94561...$$ × 12

$C = 71.016...°$, so $C = 71.0°$ sin⁻¹: the acute answer

$180° - 71.016...° = 108.98...°$ 180° − θ: the obtuse answer

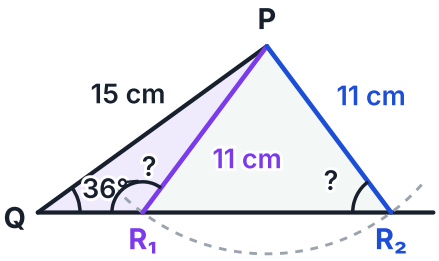
$B + C = 52° + 109.0° = 161.0° < 180°$: it fits, so $C = 109.0°$ too.

180° − θ: does it fit?



C = 71.0° or 109.0°

3 In triangle PQR, PQ = 15 cm, PR = 11 cm and angle PQR = 36°. Find the two possible values of angle PRQ.



draw a diagram: swing AC

$\frac{\sin R}{15} = \frac{\sin 36°}{11}$ PQ opposite R; PR opposite Q

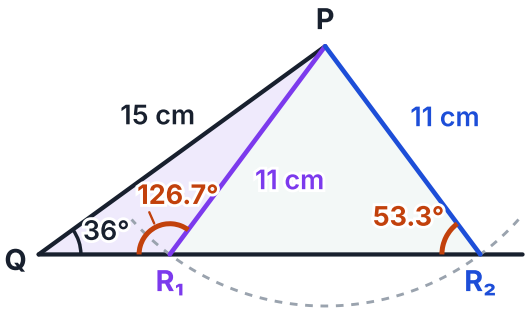
$\sin R = \frac{15 \sin 36°}{11} = 0.80152...$ × 15

$R = 53.276...°$, so $R = 53.3°$ sin⁻¹: the acute answer

$180° - 53.276...° = 126.72...°$ 180° − θ: the obtuse answer

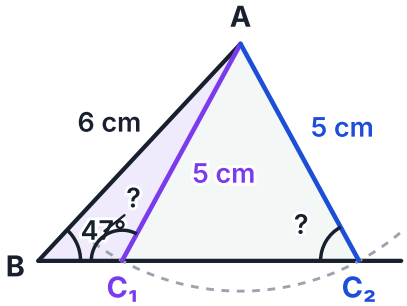
$Q + R = 36° + 126.7° = 162.7° < 180°$: it fits, so $R = 126.7°$ too.

180° − θ: does it fit?



R = 53.3° or 126.7°

4 In triangle ABC, AB = 6 cm, AC = 5 cm and angle ABC = 47°. Find the two possible values of angle ACB.



draw a diagram: swing AC

$\frac{\sin C}{6} = \frac{\sin 47°}{5}$ sine rule: sines on top

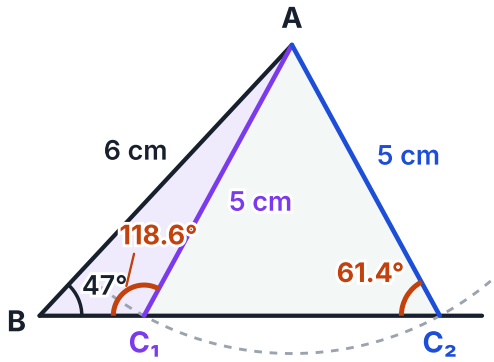
$\sin C = \frac{6 \sin 47°}{5} = 0.87762...$ × 6

$C = 61.357...°$, so $C = 61.4°$ sin⁻¹: the acute answer

$180° - 61.357...° = 118.64...°$ 180° − θ: the obtuse answer

$B + C = 47° + 118.6° = 165.6° < 180°$: it fits, so $C = 118.6°$ too.

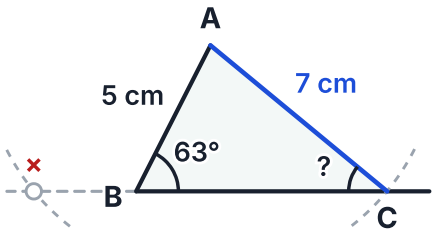
180° − θ: does it fit?



C = 61.4° or 118.6°

B · One, two or none?

5 In triangle ABC, $AB = 5$ cm, $AC = 7$ cm and angle $ABC = 63°$. Find angle ACB.



draw a diagram: swing AC

$$\frac{\sin C}{5} = \frac{\sin 63°}{7}$$ sine rule: sines on top

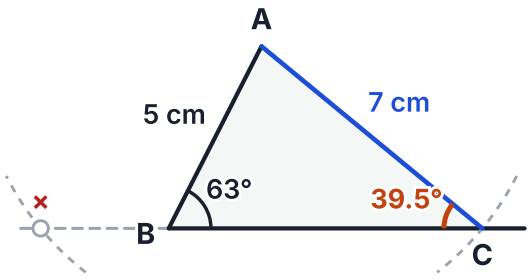
$$\sin C = \frac{5 \sin 63°}{7} = 0.63643\ldots$$ × 5

$C = 39.526\ldots°$, so $C = 39.5°$ sin⁻¹: the acute answer

$180° - 39.526\ldots° = 140.47\ldots°$ 180° − θ: the obtuse answer

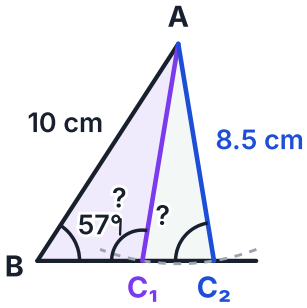
$B + C = 63° + 140.5° = 203.5° > 180°$: reject. Only $C = 39.5°$.

reject: angle sum > 180°



C = 39.5° only

6 In triangle ABC, $AB = 10$ cm, $AC = 8.5$ cm and angle $ABC = 57°$. Find all possible values of angle ACB.



draw a diagram: swing AC

$$\frac{\sin C}{10} = \frac{\sin 57°}{8.5}$$ sine rule: sines on top

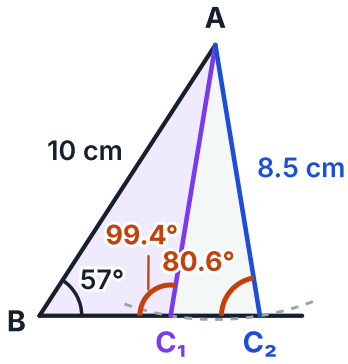
$$\sin C = \frac{10 \sin 57°}{8.5} = 0.98667\ldots$$ × 10

$C = 80.634\ldots°$, so $C = 80.6°$ sin⁻¹: the acute answer

$180° - 80.634\ldots° = 99.365\ldots°$ 180° − θ: the obtuse answer

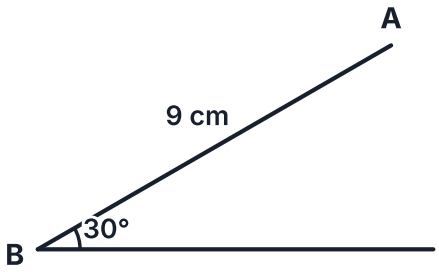
$B + C = 57° + 99.4° = 156.4° < 180°$: it fits, so $C = 99.4°$ too.

180° − θ: does it fit?



both fit: 80.6° or 99.4°

7 In triangle ABC, $AB = 9$ cm, $AC = 4$ cm and angle $ABC = 30°$. Find angle ACB.



draw what is given: AB and angle B

$\sin C = \frac{9 \sin 30°}{4} = \frac{4.5}{4} = 1.125$ sine rule: sines on top

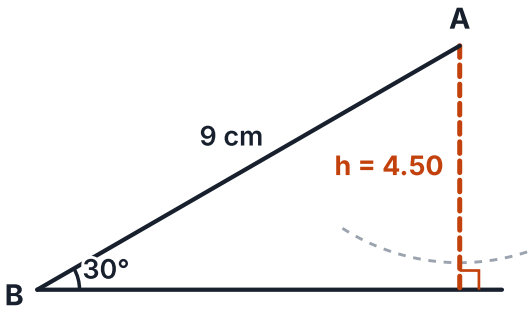
$\sin C > 1$: impossible (calculator: Math ERROR).

sin is never above 1

Reason: $h = 9 \sin 30° = 4.5$ and $AC = 4 < 4.5$: AC cannot reach the base line. No triangle.

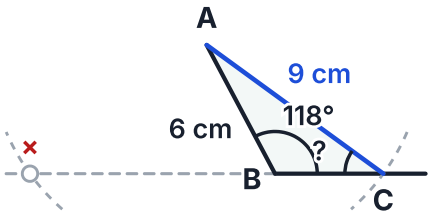
b < h: no triangle

4 < 4.50: AC cannot reach



no triangle: 4 < h = 4.5

8 In triangle ABC, $AB = 6$ cm, $AC = 9$ cm and angle $ABC = 118°$. Find angle ACB.



draw a diagram: swing AC

$\frac{\sin C}{6} = \frac{\sin 118°}{9}$ sine rule: sines on top

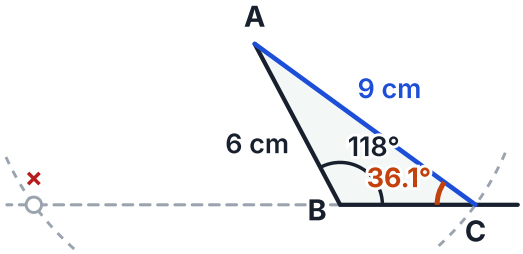
$\sin C = \frac{6 \sin 118°}{9} = 0.58863...$ × 6

$C = 36.059...°$, so $C = 36.1°$ $\sin^{-1}$: the acute answer

$180° - 36.059...° = 143.94...°$ 180° − θ: the obtuse answer

$B + C = 118° + 143.9° = 261.9° > 180°$: reject. Only $C = 36.1°$.

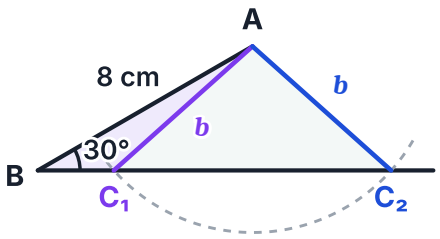
reject: angle sum > 180°



C = 36.1° only

C · Reasoning No calculator for this section.

9 In triangle ABC, AB = 8 cm, angle ABC = 30° and $AC = b$ cm.
(a) Find the least possible value of b.
(b) How many triangles are there when b = 3, 4, 6, 8 and 9?



draw a diagram: swing AC

(a) $h = 8 \sin 30° = 8 \times ½ = 4$, so the least b is 4 cm.

shortest distance: c sin B

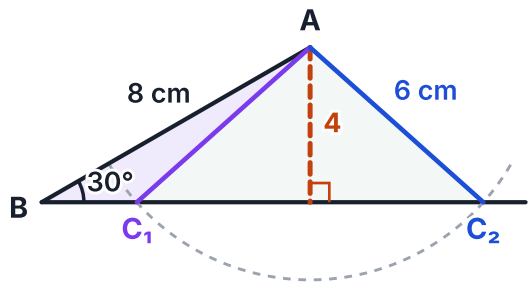
(b) $b = 3$: none (3 < 4). $b = 4$: one (right angle).

compare b with h

$b = 6$: two (4 < 6 < 8). $b = 8$: one ($b = c$; isosceles). $b = 9$: one.

compare b with c

4 < 6 < 8: two triangles



least b = 4 cm

10 (a) Write down the obtuse angle whose sine equals $\sin 35°$.
(b) $\sin x = \sin 72°$ and $x \neq 72°$, with $0° < x < 180°$. Find x.
(c) Explain why the cosine rule never gives two possible angles.

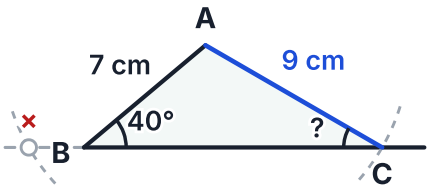
(a) $180° - 35° = 145°$ — sin θ = sin(180° − θ)

(b) $x = 180° - 72° = 108°$ — sin θ = sin(180° − θ)

(c) $\cos(180° - \theta) = -\cos\theta$: the obtuse angle has a **negative** cosine, so $\cos^{-1}$ gives exactly one angle between 0° and 180°.

$\cos^{-1}$ (negative → obtuse)

11 In triangle *ABC*, *AB* = 7 cm, *AC* = 9 cm and angle *ABC* = 40°. Without finding it, explain why angle *ACB* has only one possible value.



draw a diagram: swing AC

The larger angle is opposite the larger side:
$AB = 7 < AC = 9$, so $C < B = 40°$.

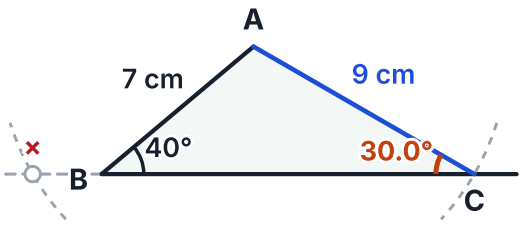
side ↔ opposite angle

The obtuse candidate would give
$B + C > 40° + 140° = 180°$: impossible.

180° − θ: does it fit?

So C is acute: one value. ($b \geq c$: one triangle.)

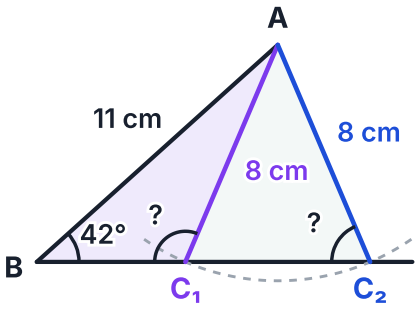
b ≥ c: one triangle



C < 40°: acute, one triangle

D · Multi-step and modelling

12 In triangle ABC, AB = 11 cm, AC = 8 cm and angle ABC = 42°.
(a) Find the two possible values of angle ACB.
(b) Find the two possible lengths of BC.
(c) Find the difference between them.



draw a diagram: swing AC

(a) $\dfrac{\sin C}{11} = \dfrac{\sin 42°}{8}$ — sine rule: sines on top

$C = 66.934...°$, so $C = 66.9°$ — $\sin^{-1}$: the acute answer

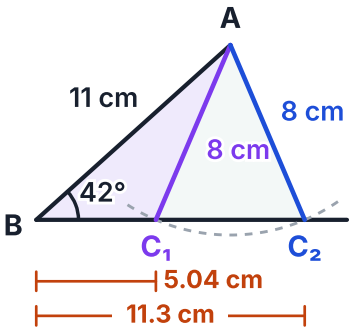
$180° - 66.934...° = 113.06...°$ — 180° − θ: the obtuse answer

$B + C = 42° + 113.1° = 155.1° < 180°$: it fits, so $C = 113.1°$ too. — 180° − θ: does it fit?

(b) $A = 180° - 42° - C = 71.065...°$ or $24.934...°$ — angles add to 180°

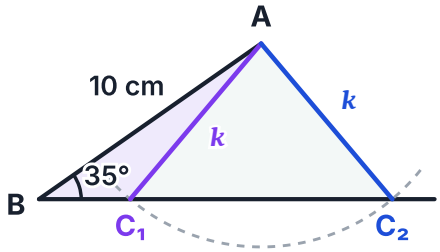
$BC = \dfrac{8 \sin A}{\sin 42°} = 11.3$ or 5.04 cm — sine rule: pair 8 with 42°

(c) 11.308... − 5.0402... = 6.27 cm — subtract full values



BC = 5.04 or 11.3 cm; difference 6.27 cm

13 In triangle ABC, AB = 10 cm, angle ABC = 35° and $AC = k$ cm. There are two possible triangles. Find the set of possible values of k. Give the lower limit exactly and to 3 s.f.



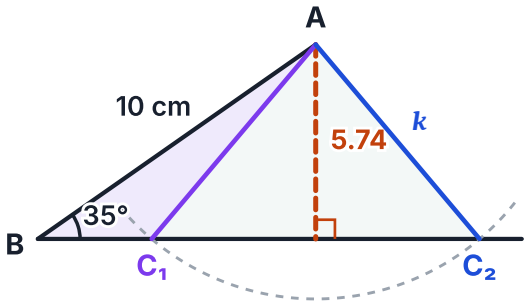
draw a diagram: swing AC between h and c

Two triangles when $h < k < c$.

$h = 10 \sin 35° = 5.7357...$ — shortest distance: c sin B

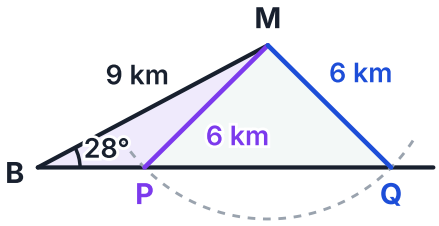
$10 \sin 35° < k < 10$, i.e. $5.74 < k < 10$ — strict: k = h gives one

5.74 < k < 10



5.74 < k < 10

14 A straight road passes through point B. A phone mast M is 9 km from B, and the angle between BM and the road is 28°. The mast has a range of 6 km. Find the length of road that is within range of the mast.



draw a diagram: road, mast, range

The road is in range between the two points P and Q that are 6 km from M (two triangles, as $h < 6 < 9$).

two crossings: h < b < c

$h = 9 \sin 28° = 4.2252...$ shortest distance: c sin B

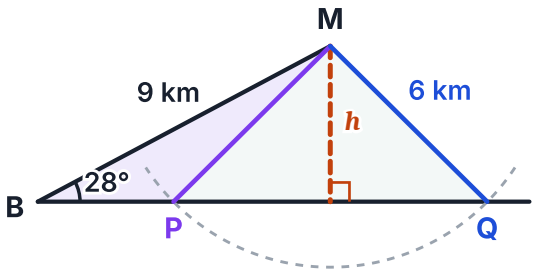
Half the length: $\sqrt{6^2 - h^2} = 4.2599...$

Pythagoras in the isosceles triangle

Length in range $= 2 \times 4.2599... = 8.5199...$, so 8.52 km

× 2; 3 s.f.

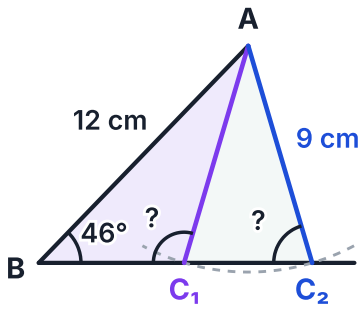
PQ = 8.52 km



road in range: 8.52 km

E · Exam-style question Use the mark scheme to self-assess.

15 In triangle ABC, $AB = 12$ cm, $AC = 9$ cm and angle $ABC = 46°$.
(a) Find the two possible values of angle ACB, showing that both are possible. [3 marks]
(b) Given that angle ACB is obtuse, find BC. [3 marks]



draw a diagram: swing AC

(a) $\sin C = \dfrac{12 \sin 46°}{9} = 0.95911...$ **M1**

sine rule: sines on top

$C = 73.6°$ or $180° - 73.6° = 106.4°$ **A1**

$\sin^{-1}$, then $180° - \theta$

$46° + 106.4° = 152.4° < 180°$: both possible **B1**

$180° - \theta$: does it fit?

(b) $A = 180° - 46° - 106.43...° = 27.560...°$ **M1**

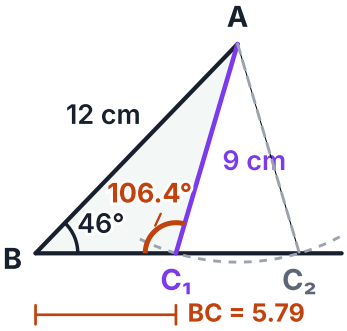
angles add to 180°

$BC = \dfrac{9 \sin 27.560...°}{\sin 46°}$ **M1**

sine rule: pair 9 with 46°

$BC = 5.7888...$, so $BC = 5.79$ cm **A1**

3 s.f., with units



(a) 73.6° or 106.4° (b) BC = 5.79 cm