Area of a Triangle

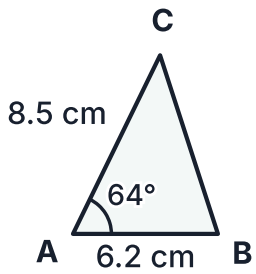
Specification: AQA 7357: E1 · Edexcel 9MA0: 5.1

INDEPENDENT

You do

A · Two sides and the included angle

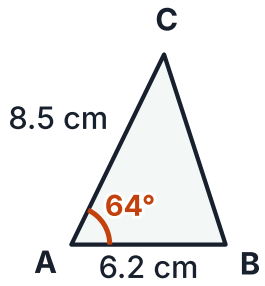
1 In triangle ABC, AB = 6.2 cm, AC = 8.5 cm and angle BAC = 64°. Find the area.



draw a diagram first

Area = ½ × 6.2 × 8.5 × sin 64°
= 23.683..., so 23.7 cm²

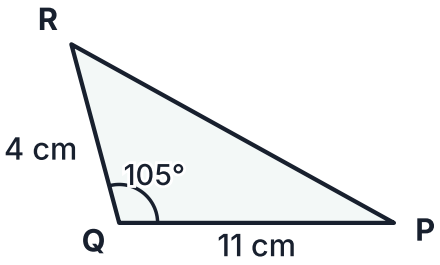
½ab sin C
calculator, 3 s.f.



Area = 23.7 cm²

area 23.7 cm²

2 In triangle PQR, PQ = 11 cm, QR = 7.4 cm and angle PQR = 105°. Find the area.

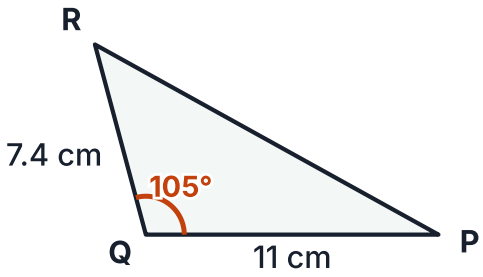


draw a diagram first

Angle Q is between PQ and QR: area
= ½ × 11 × 7.4 × sin 105°

= 39.313..., so 39.3 cm²

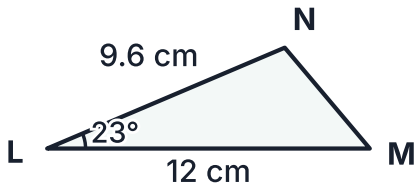
the included angle
calculator, 3 s.f.



Area = 39.3 cm²

area 39.3 cm²

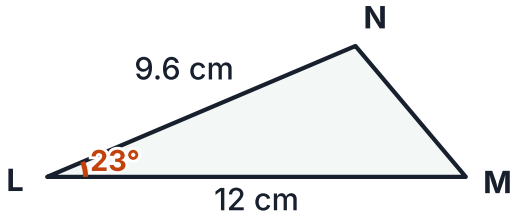
3 In triangle LMN, $LM = 12$ cm, $LN = 9.6$ cm and angle $MLN = 23°$. Find the area.



draw a diagram first

Area $= \frac{1}{2} \times 12 \times 9.6 \times \sin 23°$ — ½ab sin C

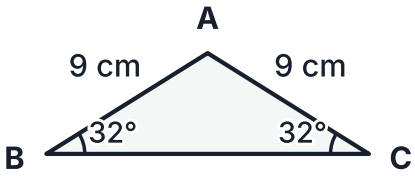
$= 22.506\ldots$, so 22.5 cm² — calculator, 3 s.f.



Area = 22.5 cm²

area 22.5 cm²

4 An isosceles triangle has two sides of 9 cm and base angles of 32°. Find its area.

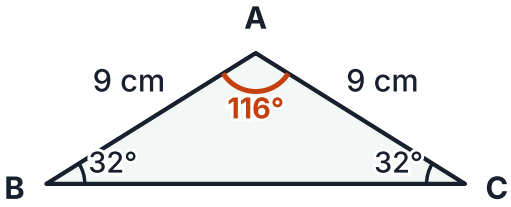


draw a diagram first

Angle between the equal sides $= 180° - 2 \times 32° = 116°$ — angles add to 180°

Area $= \frac{1}{2} \times 9 \times 9 \times \sin 116°$ — ½ab sin C

$= 36.401\ldots$, so 36.4 cm² — calculator, 3 s.f.

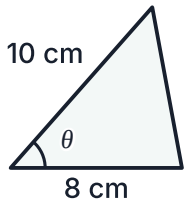


Area = 36.4 cm²

apex 116°: area 36.4 cm²

B · From the area

5 Two sides of a triangle are 8 cm and 10 cm, and the angle θ between them is acute. The area is 30 cm². Find θ.



draw a diagram first

area 30 cm²

$\frac{1}{2} \times 8 \times 10 \times \sin\theta = 30$ — ½ab sin C: substitute

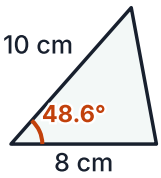
$\sin\theta = \frac{2 \times 30}{8 \times 10} = 0.75$ — rearrange for sin θ

$\theta = 48.590...°$ — $\sin^{-1}$: the acute answer

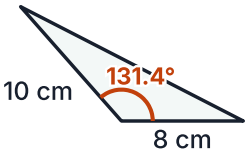
$180° - 48.590...° = 131.40...°$ has the same sine, so gives the same area.

180° − θ: does it fit?

Given acute: $\theta = 48.6°$ — given acute: reject 180° − θ



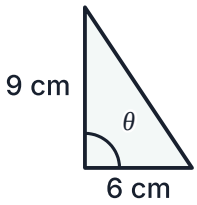
θ = 48.6° ✓



θ = 131.4° ×

acute: θ = 48.6°

6 Two sides of a triangle are 6 cm and 9 cm, with angle θ between them. The area is 18 cm². Find the two possible values of θ.



draw a diagram first

area 18 cm²

$\frac{1}{2} \times 6 \times 9 \times \sin\theta = 18$ — ½ab sin C: substitute

$\sin\theta = \frac{2 \times 18}{6 \times 9} = 0.66666...$ — rearrange for sin θ

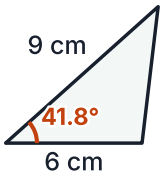
$\theta = 41.810...°$ — $\sin^{-1}$: the acute answer

$180° - 41.810...° = 138.18...°$ has the same sine, so gives the same area.

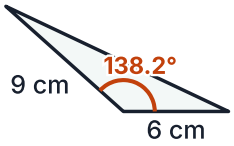
180° − θ: does it fit?

Only one angle of the triangle is known, so both fit:
$\theta = 41.8°$ or $138.2°$

both fit: angle sum < 180°



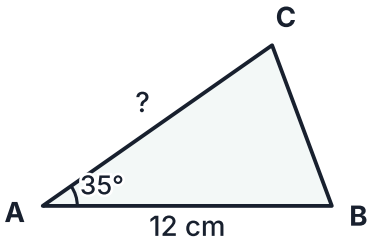
θ = 41.8° ✓



θ = 138.2° ✓

both: θ = 41.8° or 138.2°

7 In triangle ABC, AB = 12 cm and angle BAC = 35°. The area is 40 cm². Find AC.



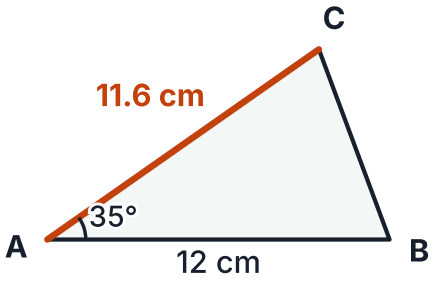
draw a diagram first

area 40 cm²

$\frac{1}{2} \times 12 \times AC \times \sin 35° = 40$ — ½ab sin C: substitute

$AC = \dfrac{80}{12 \sin 35°} = 11.622\ldots$ — rearrange for AC

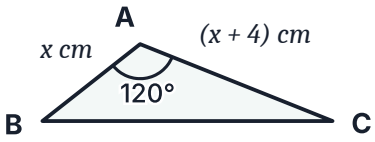
$AC = 11.6$ cm — 3 s.f.



Area = 40 cm²

AC = 11.6 cm

8 No calculator. In triangle ABC, AB = x cm, AC = (x + 4) cm and angle BAC = 120°. The area is 15√3 cm². Find x.



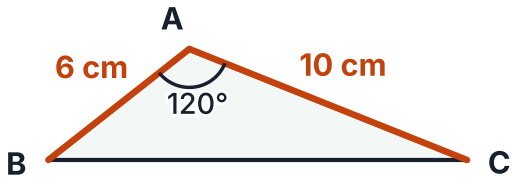
draw a diagram first

area 15√3 cm²

$\frac{1}{2}x(x+4) \times \dfrac{\sqrt{3}}{2} = 15\sqrt{3}$ — sin 120° = √3/2

$x(x+4) = 60 \Rightarrow x^2 + 4x - 60 = 0$ — ÷ √¾; rearrange

$(x+10)(x-6) = 0$, so $x = 6$ — factorise; a length is positive

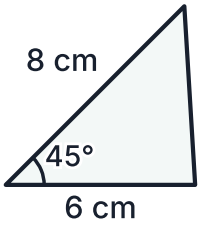


x = 6

Area = 15√3 cm²

C · Reasoning No calculator for this section.

9 Two sides of a triangle are 6 cm and 8 cm.
(a) Find the exact area when the angle between them is 45°.
(b) Find the exact area when the angle between them is 135°.
(c) Explain why the answers are the same.



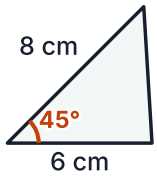
draw a diagram first

(a) $\frac{1}{2} \times 6 \times 8 \times \frac{\sqrt{2}}{2} = 12\sqrt{2}$ cm² — sin 45° = √⅔

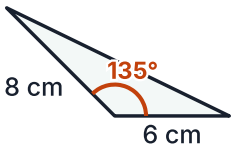
(b) $\sin 135° = \sin 45°$, so the area is also $12\sqrt{2}$ cm² — sin θ = sin(180° − θ)

(c) 135° = 180° − 45°: the two angles have the same sine, so two different triangles have the same area.

same sine, same area



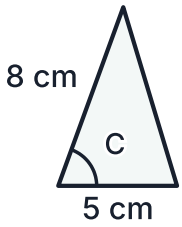
θ = 45° ✓



θ = 135° ✓

same area 12√2 cm²

10 Two sides of a triangle are 5 cm and 8 cm. Explain why the area can never be more than 20 cm². Which angle gives exactly 20 cm²?

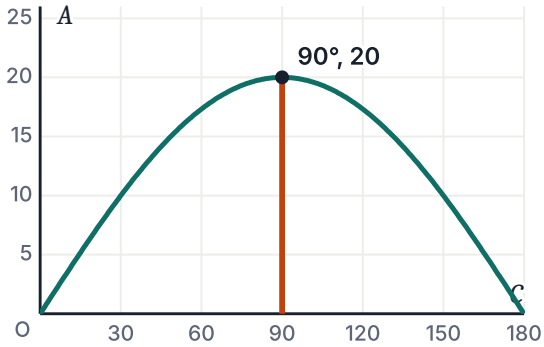


draw a diagram first

Area = $\frac{1}{2} \times 5 \times 8 \times \sin C = 20 \sin C$ — ½ab sin C

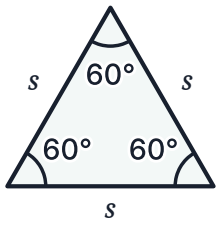
$\sin C \le 1$, so the area is at most 20 cm². — sin C ≤ 1

Area 20 only when $\sin C = 1$: $C = 90°$. — largest area: C = 90°



area = 20 sin C: largest 20 at C = 90°

11 Show that an equilateral triangle with sides of length s has area $\frac{\sqrt{3}}{4}s^2$.



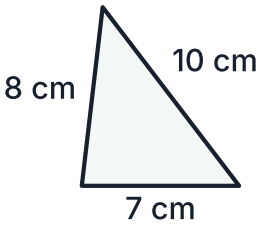
draw a diagram first

Every angle is 60°: area = $\frac{1}{2} \times s \times s \times \sin 60°$ — equilateral: 60°

$= \frac{1}{2}s^2 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}s^2$ — sin 60° = √⅔

D · Multi-step and modelling

12 A triangle has sides 7 cm, 8 cm and 10 cm. Find its area.



draw a diagram first

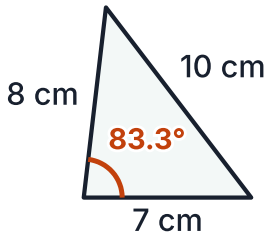
Find an angle first (cosine rule). Angle θ between 7 and 8: $\cos\theta = \dfrac{7^2 + 8^2 - 10^2}{2 \times 7 \times 8} = \dfrac{13}{112}$

cosine rule: rearrange for cos

$\theta = 83.334...°$ $\cos^{-1}$

Area = ½ × 7 × 8 × sin 83.334...° = 27.810..., so 27.8 cm²

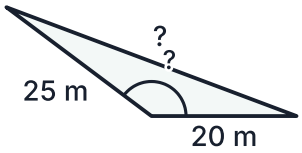
½ab sin C



Area = 27.8 cm²

angle 83.3°; area 27.8 cm²

13 A triangular garden has area 150 m². Two of its fences are 20 m and 25 m long, and the angle between them is obtuse.
(a) Find the angle between these two fences.
(b) Find the total length of fencing around the garden.



draw a diagram first

area 150 m²

(a) ½ × 20 × 25 × sin θ = 150 ½ab sin C: substitute

$\sin\theta = \dfrac{2 \times 150}{20 \times 25} = 0.6$ rearrange for sin θ

$\theta = 36.869...°$ $\sin^{-1}$: the acute answer

Given obtuse: $\theta = 180° - 36.869...° = 143.13...°$, so $\theta = 143.1°$

given obtuse: 180° − θ

(b) Third fence:

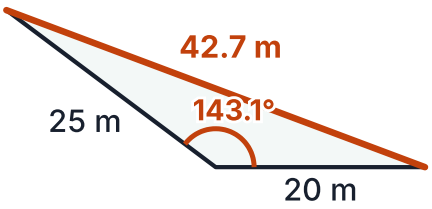
$x^2 = 20^2 + 25^2 - 2(20)(25)\cos 143.13...° = 1824.9...$

cosine rule

$x = 42.720...$ m square root

Total = 20 + 25 + 42.720... = 87.720..., so 87.7 m

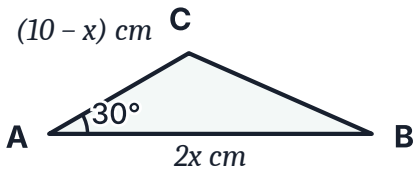
add the three; 3 s.f.



Area = 150 m²

angle 143.1°; fencing 87.7 m

14 In triangle ABC, $AB = 2x$ cm, $AC = (10 - x)$ cm and angle $BAC = 30°$. The area is A cm².
(a) Show that $A = \frac{1}{2}(10x - x^2)$.
(b) Find the maximum area and the lengths of AB and AC when it occurs.



draw a diagram first

(a) $A = \frac{1}{2} \times 2x \times (10 - x) \times \frac{1}{2} = \frac{1}{2}x(10 - x) = \frac{1}{2}(10x - x^2)$

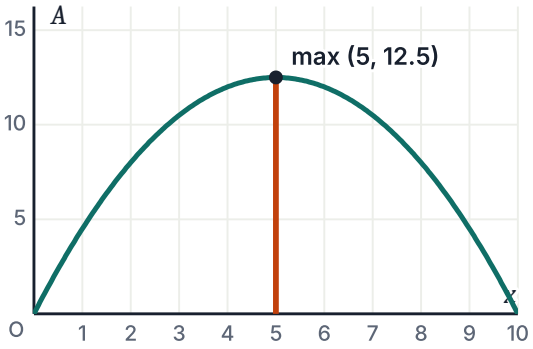
½ab sin C; sin 30° = ½

(b) $10x - x^2 = 25 - (x - 5)^2$ complete the square

Maximum $A = \frac{25}{2} = 12.5$ cm², at $x = 5$

largest when the square is 0

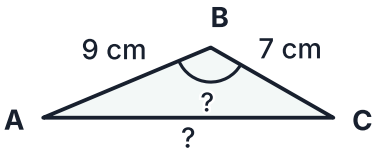
Then $AB = 10$ cm and $AC = 5$ cm. substitute x = 5



A is largest (12.5) at x = 5

E · Exam-style question Use the mark scheme to self-assess.

15 In triangle ABC, $AB = 9$ cm, $BC = 7$ cm and the area of the triangle is 25 cm². Angle ABC is obtuse.
(a) Find angle ABC. [3 marks]
(b) Find AC. [3 marks]



draw a diagram first

area 25 cm²

(a) $\frac{1}{2} \times 9 \times 7 \times \sin B = 25 \Rightarrow \sin B = \frac{50}{63} = 0.79365\ldots$ **M1** — ½ab sin C: rearrange

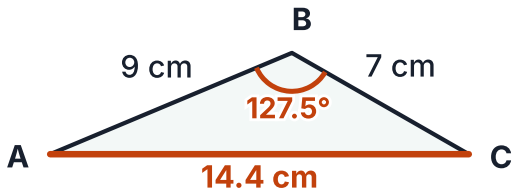
$\sin^{-1}$ gives $52.528\ldots°$; obtuse, so $B = 180° - 52.528\ldots°$ **M1** — given obtuse: 180° − θ

$B = 127.5°$ **A1** — 1 d.p.

(b) $AC^2 = 9^2 + 7^2 - 2(9)(7)\cos 127.47\ldots°$ **M1** — cosine rule

$AC^2 = 206.65\ldots$ **A1** — cos is negative: add

$AC = 14.375\ldots$, so $AC = 14.4$ cm **A1** — 3 s.f., with units



B = 127.5°, AC = 14.4 cm

Area = 25 cm²