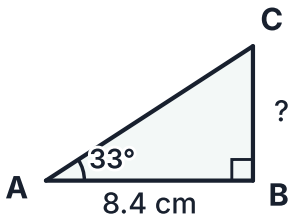
Solving Triangle Problems

Specification: AQA 7357: E1 · Edexcel 9MA0: 5.1

INDEPENDENT **You do**

A · Which rule?

1 In triangle ABC, angle $ABC = 90°$, $AB = 8.4$ cm and angle $BAC = 33°$. Find BC.



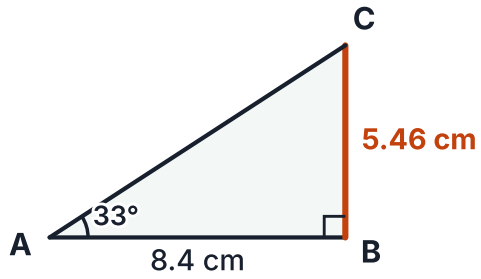
draw a diagram first

Right angle: right-angled trig. BC is opposite 33°, AB is adjacent.

right angle: opp, adj

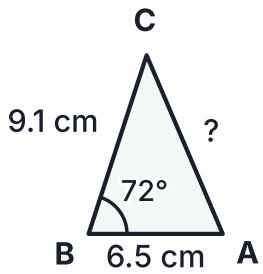
$BC = 8.4 \tan 33° = 5.4550...$, so $BC = 5.46$ cm

tan = opp ÷ adj; 3 s.f.



BC = 5.46 cm

2 In triangle ABC, $AB = 6.5$ cm, $BC = 9.1$ cm and angle $ABC = 72°$. Find AC.



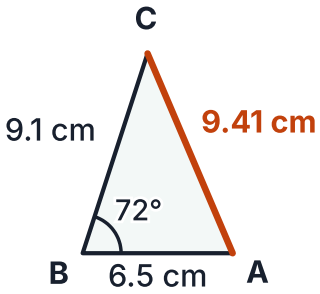
draw a diagram first

Two sides and the angle between them: the cosine rule.

which rule? cosine rule

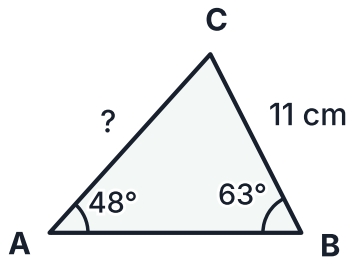
$AC^2 = 6.5^2 + 9.1^2 - 2(6.5)(9.1)\cos 72° = 88.503...$ substitute

$AC = 9.4076...$, so $AC = 9.41$ cm square root; 3 s.f.



AC = 9.41 cm

3 In triangle ABC, angle $BAC = 48°$, angle $ABC = 63°$ and $BC = 11$ cm. Find AC.



draw a diagram first

BC and A are an opposite pair; AC is opposite B: the sine rule.

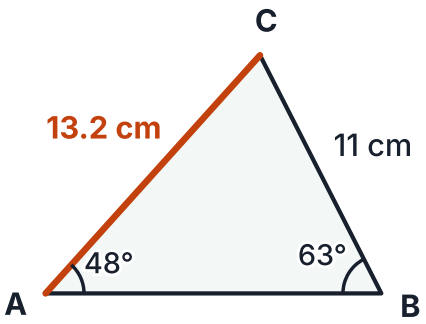
which rule? sine rule

$$\frac{AC}{\sin 63°} = \frac{11}{\sin 48°} \Rightarrow AC = 13.188\ldots$$

rearrange

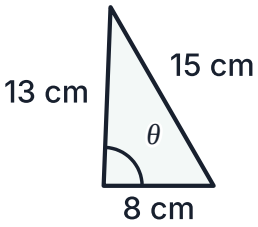
$AC = 13.2$ cm

3 s.f.



AC = 13.2 cm

4 A triangle has sides 8 cm, 13 cm and 15 cm. Find its largest angle.



draw a diagram first

Check: $8 + 13 > 15$, so it is a triangle. Three sides → the cosine rule; the largest angle is opposite 15 cm.

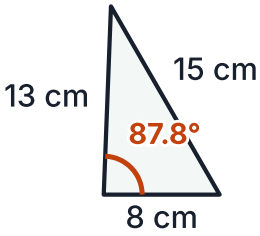
check, then cosine rule

$$\cos\theta = \frac{8^2 + 13^2 - 15^2}{2 \times 8 \times 13} = \frac{8}{208}$$

rearrange for cos θ

$\theta = 87.8°$

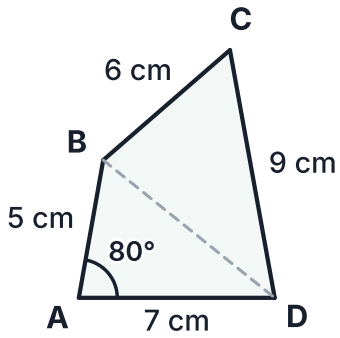
$\cos^{-1}$; 1 d.p.



largest angle 87.8°

B · Multi-step problems

5 The diagram shows quadrilateral *ABCD*.
(a) Find the length of *BD*.
(b) Find angle *BCD*.
(c) Find the area of *ABCD*.

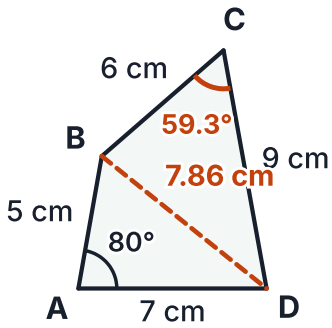


(a) Triangle ABD: $BD^2 = 5^2 + 7^2 - 2(5)(7)\cos 80° = 61.844\ldots$ — cosine rule: the shared side
$BD = 7.8641\ldots$, so $BD = 7.86$ cm — keep the full value

(b) Triangle BCD: $\cos C = \dfrac{6^2 + 9^2 - 7.8641\ldots^2}{2 \times 6 \times 9} = 0.5106\ldots$ — three sides: rearrange for cos

$C = 59.3°$ — $\cos^{-1}$; 1 d.p.

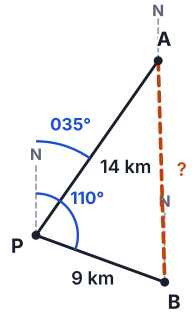
(c) Area $= \frac{1}{2}(5)(7)\sin 80° + \frac{1}{2}(6)(9)\sin 59.289\ldots° = 40.447\ldots$, so 40.4 cm² — ½ab sin C twice; add



BD = 7.86 cm, BCD = 59.3°

area 40.4 cm²

6 Two ships leave port P. Ship A sails 14 km on a bearing of 035°. Ship B sails 9 km on a bearing of 110°.
(a) Find the distance AB.
(b) Find the bearing of B from A.



draw a diagram: North lines first

(a) Angle $APB = 110° - 035° = 75°$ (both bearings are measured from the same North line at P).

angle between the paths

$AB^2 = 14^2 + 9^2 - 2(14)(9)\cos 75° = 211.77...$ cosine rule

$AB = 14.552...$, so $AB = 14.6$ km 3 s.f.

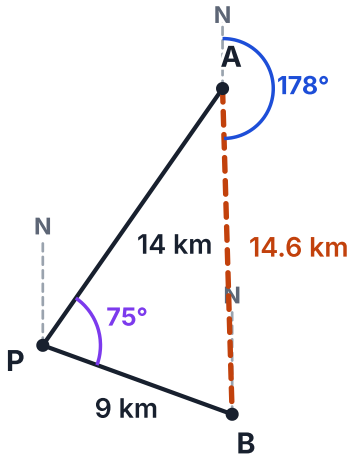
(b) $\sin PAB = \dfrac{9 \sin 75°}{14.552...} \Rightarrow PAB = 36.682...°$ (acute: opposite the shortest side)

sine rule: angle at A

Bearing of P from A: $035° + 180° = 215°$ back bearing

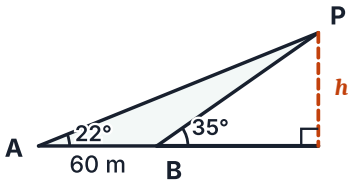
Bearing of B from A: $215° - 36.682...° = 178.3°$, so 178°

turn from AP towards B



AB = 14.6 km, bearing 178°

7 A and B are 60 m apart on level ground, in a straight line with the foot of a mast, with B nearer the mast. The angles of elevation of the top of the mast from A and B are 22° and 35°. Find the height of the mast.



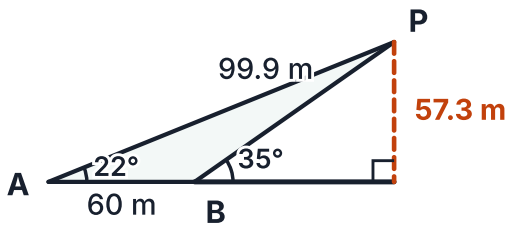
draw a diagram first

Angle $ABP = 180° - 35° = 145°$, angle $APB = 35° - 22° = 13°$

angles in triangle ABP

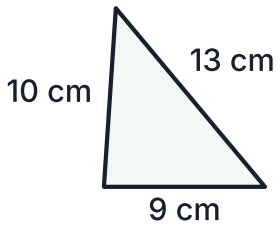
$BP = \dfrac{60 \sin 22°}{\sin 13°} = 99.916...$ sine rule

$h = BP \sin 35° = 57.309...$, so 57.3 m right-angled triangle: sin



height 57.3 m

8 A triangle has sides 9 cm, 10 cm and 13 cm.
(a) Find its area.
(b) Find the shortest distance from the opposite vertex to the 13 cm side.



draw a diagram first

(a) Angle θ between 9 and 10:

$$\cos\theta = \frac{9^2 + 10^2 - 13^2}{2 \times 9 \times 10} = \frac{12}{180}$$

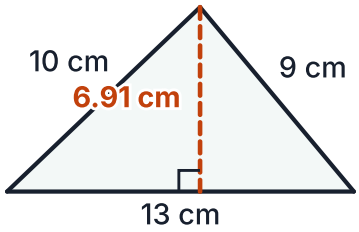
cosine rule: an angle first

$\theta = 86.177...°$; area = ½ × 9 × 10 × sin 86.177...° = 44.899..., so 44.9 cm²

½ab sin C

(b) ½ × 13 × h = 44.899... ⇒ h = 6.9076..., so 6.91 cm

area = ½ × base × height

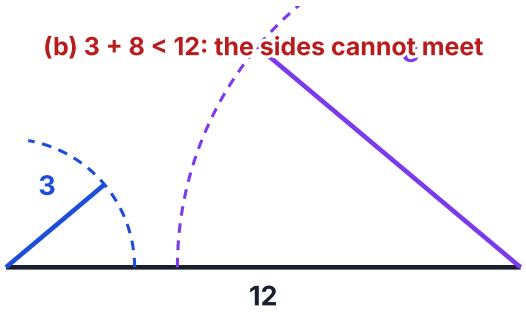


area 44.9 cm²

area 44.9 cm², h = 6.91 cm

C · Is it a triangle? No calculator for this section.

9 Which of these sets of lengths can be the sides of a triangle? Explain each answer.
(a) 4 cm, 7 cm, 10 cm
(b) 3 cm, 8 cm, 12 cm
(c) 5 cm, 5 cm, 10 cm



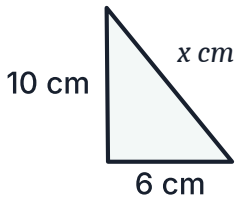
draw a diagram first

(a) $4 + 7 = 11 > 10$: yes, a triangle. shorter two > longest

(b) $3 + 8 = 11 < 12$: no, the short sides cannot meet.
shorter two < longest: no

(c) $5 + 5 = 10$: no, the sides lie flat (a straight line).
equal: flat, no triangle

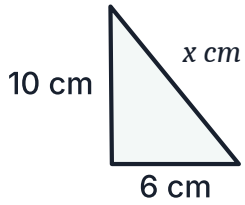
10 A triangle has sides 6 cm, 10 cm and x cm. Find the range of possible values of x.



draw a diagram first

sketch: not to scale

If x is the longest: $6 + 10 > x$, so $x < 16$ longest: x < 6 + 10
If 10 is the longest: $6 + x > 10$, so $x > 4$ longest 10: 6 + x > 10
$4 < x < 16$ combine

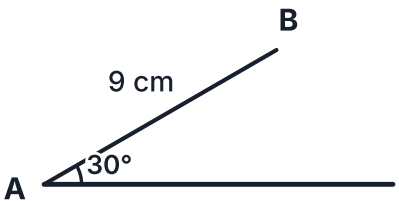


10 − 6 < x < 10 + 6

sketch: not to scale

4 < x < 16

11 Explain why there is no triangle ABC with AB = 9 cm, BC = 4 cm and angle BAC = 30°.



BC = 4 cm: can it reach?

draw a diagram first

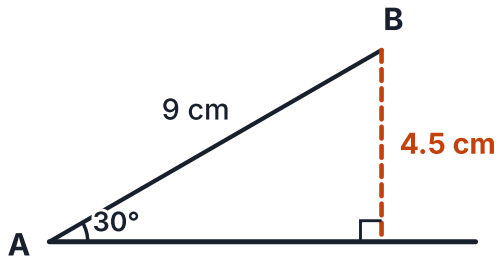
The shortest possible BC is the perpendicular from B to the line AC: $9 \sin 30° = 4.5$ cm.

shortest distance: c sin A

$4 < 4.5$, so BC cannot reach the line AC (Lesson 3).

too short: no triangle

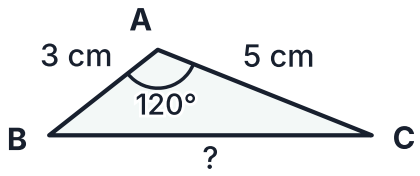
Or: $\sin C = \dfrac{9 \sin 30°}{4} = \dfrac{9}{8} > 1$, impossible. sin C > 1: none



shortest BC = 4.5 cm > 4 cm

D · Exact answers No calculator for this section.

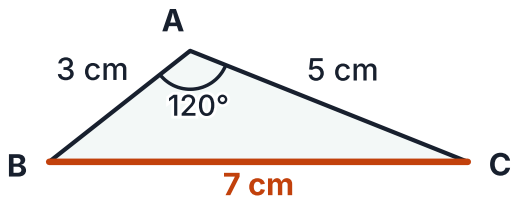
12 In triangle ABC, $AB = 3$ cm, $AC = 5$ cm and angle $BAC = 120°$.
(a) Show that $BC = 7$ cm.
(b) Find the exact area of the triangle.



draw a diagram first

(a) $BC^2 = 3^2 + 5^2 - 2(3)(5)\cos 120°$ cosine rule
$= 9 + 25 - 30 \times (-½) = 34 + 15 = 49$, so $BC = 7$ cm
cos 120° = −½

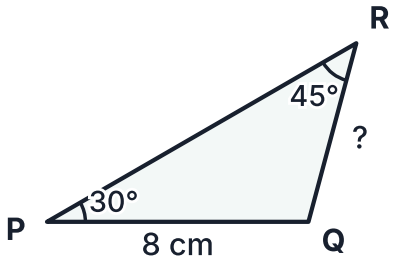
(b) Area $= ½ \times 3 \times 5 \times \frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{4}$ cm² sin 120° = √3/2



Area = 15√3 / 4 cm²

BC = 7 cm

13 In triangle PQR, $PQ = 8$ cm, angle $QPR = 30°$ and angle $PRQ = 45°$. Show that $QR = 4\sqrt{2}$ cm.



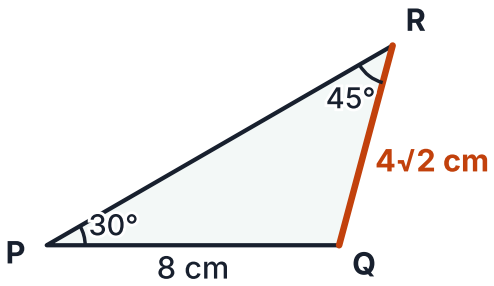
draw a diagram first

QR is opposite 30°, PQ is opposite 45°: $\frac{QR}{\sin 30°} = \frac{8}{\sin 45°}$
sine rule: opposite pairs

$QR = \frac{8 \times ½}{\frac{\sqrt{2}}{2}} = \frac{8}{\sqrt{2}}$ sin 30° = ½, sin 45° = √2/2

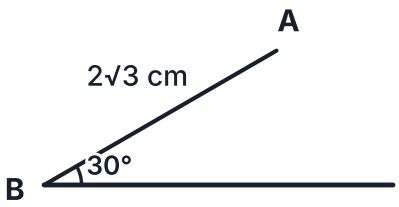
$\frac{8}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$ cm, as required

rationalise the denominator



QR = 4√2 cm

14 In triangle ABC, $AB = 2\sqrt{3}$ cm, $AC = 2$ cm and angle $ABC = 30°$.
(a) Show that angle $ACB = 60°$ or $120°$.
(b) Find the exact length of BC in each case.



AC = 2 cm: where is C?

draw a diagram first

(a) $\sin C = \dfrac{2\sqrt{3} \times \sin 30°}{2} = \dfrac{\sqrt{3}}{2}$ sine rule for the angle

$C = 60°$, or $180° - 60° = 120°$: $30° + 120° < 180°$, so both fit.

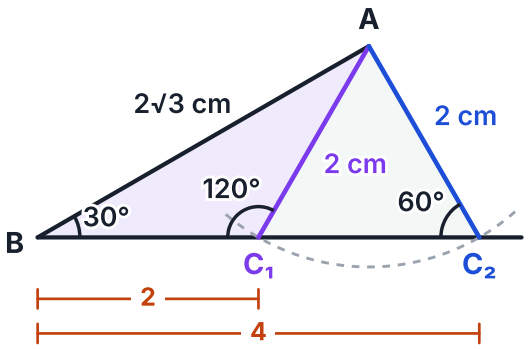
$180° - \theta$: does it fit?

(b) $C = 60°$: $A = 90°$, so $BC^2 = (2\sqrt{3})^2 + 2^2 = 16$, $BC = 4$ cm

A = 90°: Pythagoras

$C = 120°$: $A = 30° = B$, so the triangle is isosceles:
$BC = AC = 2$ cm

A = B: isosceles



BC = 4 cm (C = 60°) or 2 cm (C = 120°)

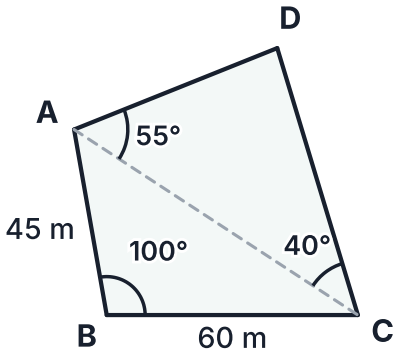
E · Exam-style question Use the mark scheme to self-assess.

15 The diagram shows a plot of land $ABCD$. AB = 45 m, BC = 60 m, angle ABC = 100°, angle CAD = 55° and angle ACD = 40°.

(a) Find the length of AC. [2 marks]

(b) Find the length of AD. [3 marks]

(c) Find the area of the plot. [3 marks]



(a) $AC^2 = 45^2 + 60^2 - 2(45)(60)\cos 100° = 6562.7...$ **M1** — cosine rule

$AC = 81.010...$, so $AC = 81.0$ m **A1** — 3 s.f.

(b) Angle $ADC = 180° - 55° - 40° = 85°$ **B1** — angles add to 180°

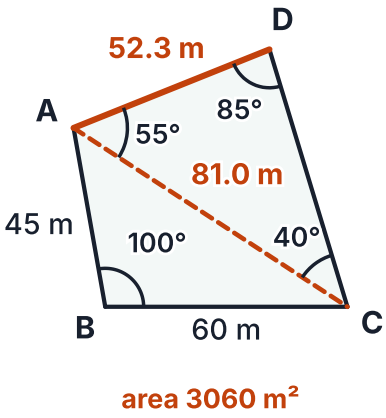
$\frac{AD}{\sin 40°} = \frac{81.010...}{\sin 85°}$ **M1** — sine rule: AD opposite 40°

$AD = 52.271...$, so $AD = 52.3$ m **A1** — 3 s.f.

(c) Area $= \frac{1}{2}(45)(60)\sin 100° + \frac{1}{2}(81.010...)(52.271...)\sin 55°$ **M1** — ½ab sin C twice

$= 1329.4... + 1734.3...$ **A1** — each triangle

$= 3063.8...$, so 3060 m² **A1** — 3 s.f., with units



AC = 81.0 m, AD = 52.3 m

area 3060 m²