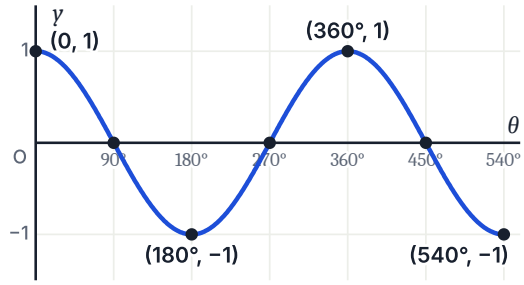
Graphs of sin, cos and tan

Specification: AQA 7357: E1, E3 · Edexcel 9MA0: 5.1, 5.3

INDEPENDENT **You do**

A · Key features

1 Sketch the graph of $y = \cos\theta$ for $0° \le \theta \le 540°$. Show where it meets the axes.



draw the axes: mark the interval

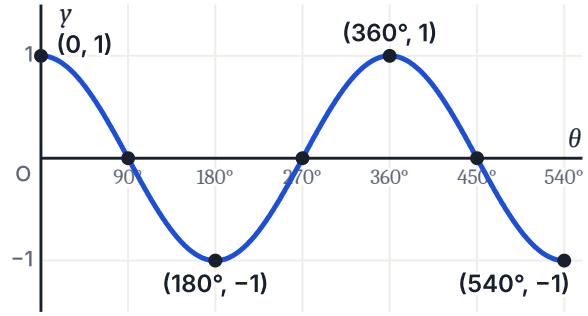
Zeros: $\theta = 90°, 270°, 450°$ — zeros: 90° + 180°n

Maximum 1 at $\theta = 0°, 360°$; minimum −1 at $\theta = 180°, 540°$

max 1, min −1

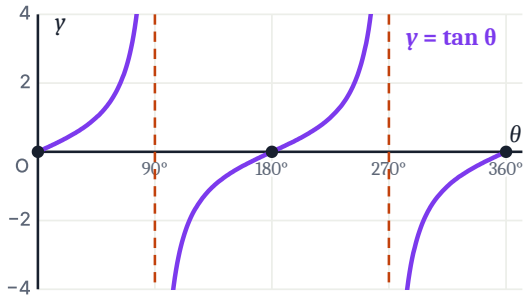
Join them with a smooth wave through (0, 1). Period 360°.

smooth wave, period 360°



y = cos θ, 0° ≤ θ ≤ 540°

2 Sketch the graph of $y = \tan\theta$ for $0° \le \theta \le 360°$. Show the asymptotes.



draw the axes: mark the interval

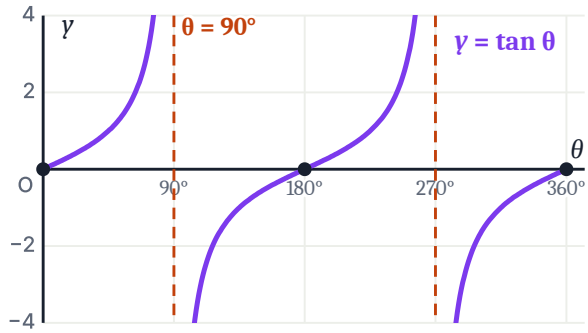
Asymptotes (tan θ undefined): $\theta = 90°, 270°$

asymptote: θ = 90° + 180°n

Zeros: $\theta = 0°, 180°, 360°$ — zeros: every 180°

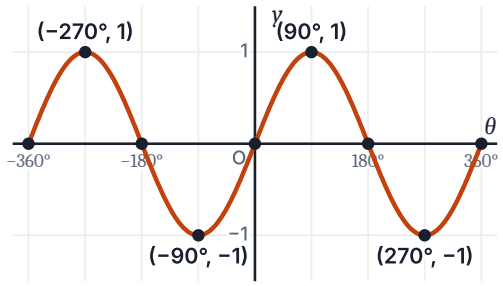
Each branch rises from $-\infty$ to $+\infty$ between two asymptotes, through a zero. Period 180°.

period 180°



asymptotes θ = 90°, 270°

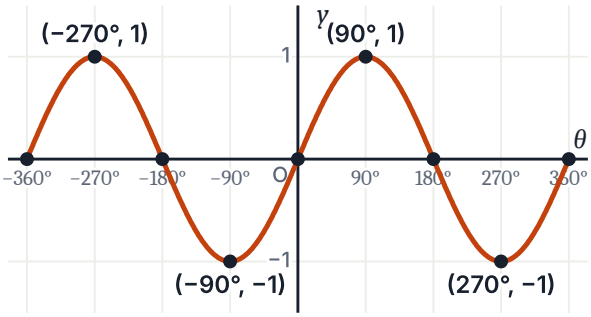
3 Write down the coordinates of every maximum point and every minimum point of $y = \sin\theta$ for $-360° \le \theta \le 360°$.



sketch the graph first

$\sin\theta = 1$ at 90°; subtract 360°: −270° — max: 90° ± 360°

$\sin\theta = -1$ at 270°; subtract 360°: −90° — min: 270° ± 360°



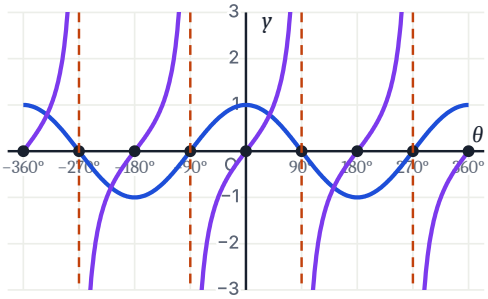
two maxima and two minima

4 For $-360° \le \theta \le 360°$, write down all the values of θ where

(a) $\cos\theta = 0$

(b) $\tan\theta = 0$

(c) $\tan\theta$ is undefined.



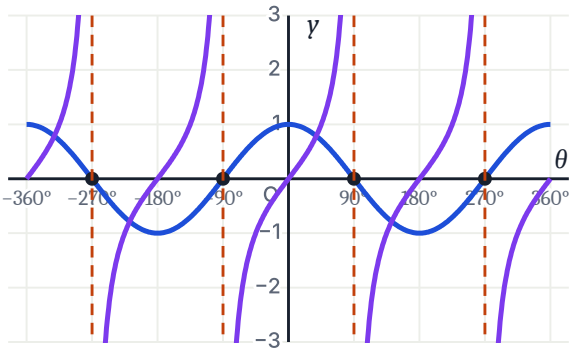
sketch both graphs first

(a) $\theta = -270°, -90°, 90°, 270°$ — zeros: 90° + 180°n

(b) $\theta = -360°, -180°, 0°, 180°, 360°$ — zeros: every 180°

(c) Where $\cos\theta = 0$: $\theta = -270°, -90°, 90°, 270°$

asymptote: θ = 90°

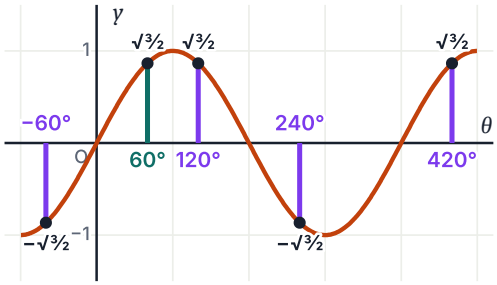


cos θ = 0 where tan θ is undefined

B · Exact values from the symmetry No calculator for this section.

5 Given that $\sin 60° = \frac{\sqrt{3}}{2}$, find the exact values of

(a) $\sin 120°$
(b) $\sin 240°$
(c) $\sin(-60°)$
(d) $\sin 420°$



sketch: mark the given point

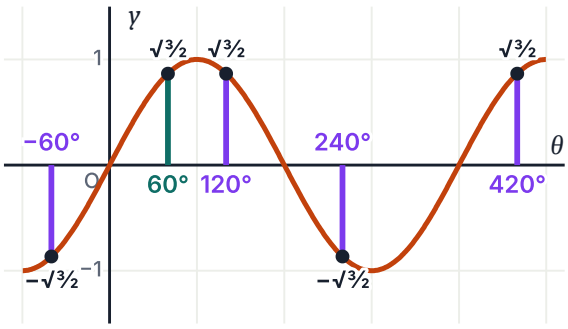
(a) $\sin 120° = \sin(180° - 60°) = \frac{\sqrt{3}}{2}$ symmetry: 180° − θ

(b) $240° = 180° - (-60°)$: $\sin 240° = \sin(-60°) = -\frac{\sqrt{3}}{2}$

180° − θ, then −θ

(c) $\sin(-60°) = -\sin 60° = -\frac{\sqrt{3}}{2}$ sin(−θ) = −sin θ

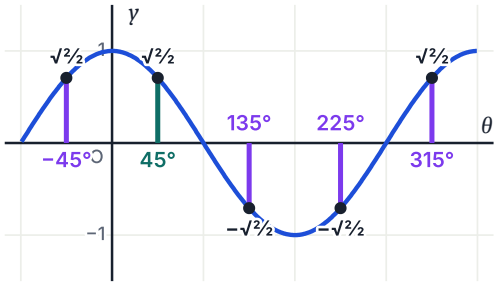
(d) $\sin 420° = \sin(60° + 360°) = \frac{\sqrt{3}}{2}$ period 360°



the given point and its images

6 Given that $\cos 45° = \frac{\sqrt{2}}{2}$, find the exact values of

(a) $\cos 315°$
(b) $\cos(-45°)$
(c) $\cos 135°$
(d) $\cos 225°$



sketch: mark the given point

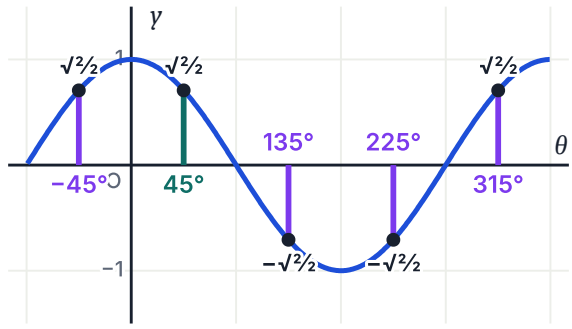
(a) $\cos 315° = \cos(360° - 45°) = \frac{\sqrt{2}}{2}$ cos(360° − θ) = cos θ

(b) $\cos(-45°) = \cos 45° = \frac{\sqrt{2}}{2}$ cos(−θ) = cos θ

(c) $\cos 135° = \cos(180° - 45°) = -\frac{\sqrt{2}}{2}$ cos(180° − θ) = −cos θ

(d) $\cos 225° = \cos(360° - 225°) = \cos 135° = -\frac{\sqrt{2}}{2}$

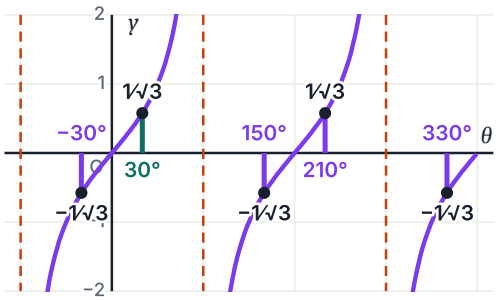
cos(360° − θ), then (c)



the given point and its images

7 Given that $\tan 30° = \frac{1}{\sqrt{3}}$, find the exact values of

(a) $\tan 210°$
(b) $\tan(-30°)$
(c) $\tan 150°$
(d) $\tan 330°$



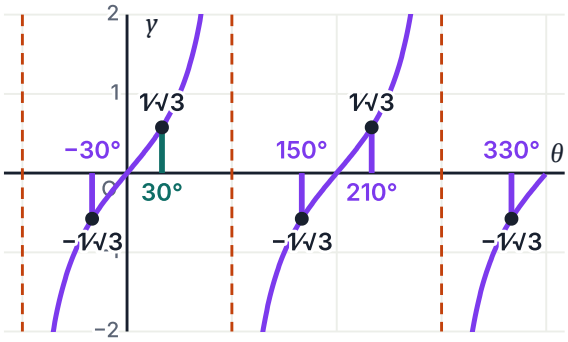
sketch: mark the given point

(a) $\tan 210° = \tan(30° + 180°) = \frac{1}{\sqrt{3}}$ — period 180°

(b) $\tan(-30°) = -\tan 30° = -\frac{1}{\sqrt{3}}$ — $\tan(-\theta) = -\tan\theta$

(c) $\tan 150° = \tan(-30° + 180°) = -\frac{1}{\sqrt{3}}$ — period 180°, then (b)

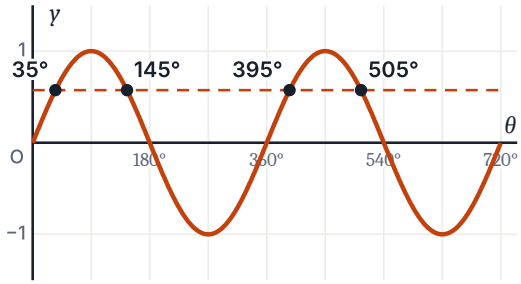
(d) $\tan 330° = \tan(-30° + 360°) = -\frac{1}{\sqrt{3}}$ — period 180° twice



the given point and its images

C · Other angles from the graph

8 Find all the values of θ in the interval $0° \le \theta \le 720°$ for which $\sin\theta = \sin 35°$.



sketch the graph and the line

One value: $\theta = 35°$ — the given angle

Symmetry of the graph: $180° - 35° = 145°$

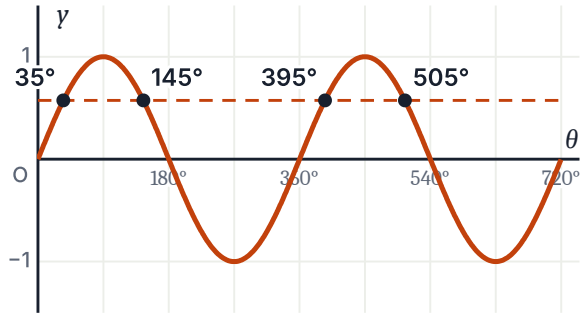
symmetry: 180° − θ

Add or subtract 360° until you leave the interval:
$\theta = 35°, 145°, 395°, 505°$

period 360°: ± 360°

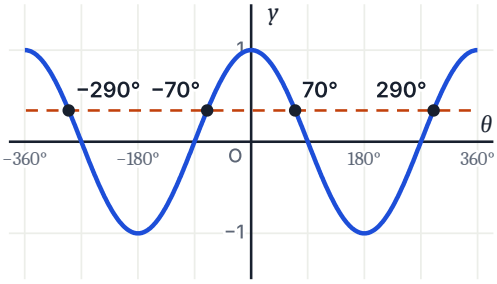
Check: the line meets the graph 4 times in $0° \le \theta \le 720°$.

count the crossings



35°, 145°, 395°, 505°

9 Find all the values of θ in the interval $-360° \le \theta \le 360°$ for which $\cos\theta = \cos 70°$.



sketch the graph and the line

One value: $\theta = 70°$ — the given angle

Symmetry in the y-axis: $\cos(-70°) = \cos 70°$, so $\theta = -70°$

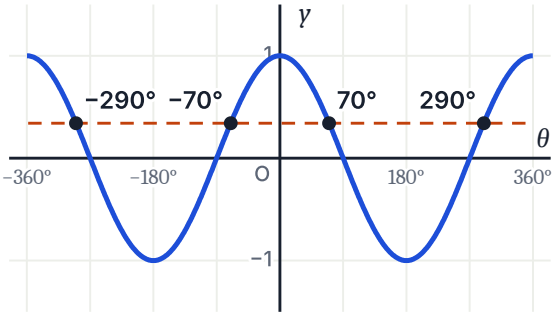
symmetry: cos(−θ) = cos θ

Add or subtract 360° until you leave the interval:
$\theta = -290°, -70°, 70°, 290°$

period 360°: ± 360°

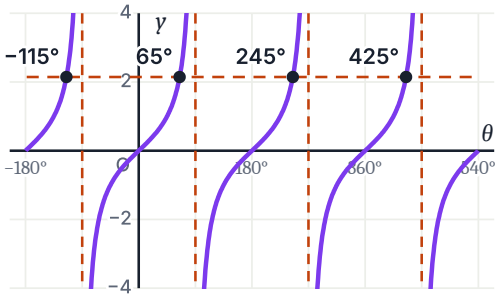
Check: the line meets the graph 4 times in $-360° \le \theta \le 360°$.

count the crossings



−290°, −70°, 70°, 290°

10 Find all the values of θ in the interval $-180° \le \theta \le 540°$ for which $\tan\theta = \tan 65°$.



sketch the graph and the line

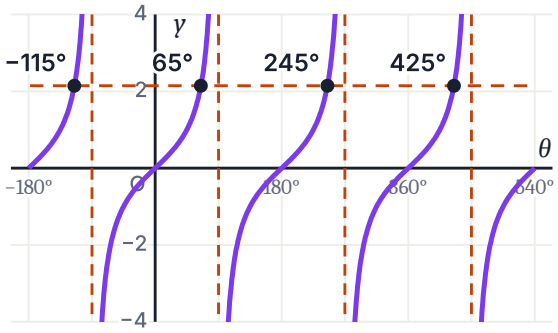
One value: $\theta = 65°$ — the given angle

Add or subtract 180° until you leave the interval:
$\theta = -115°, 65°, 245°, 425°$

period 180°: ± 180°

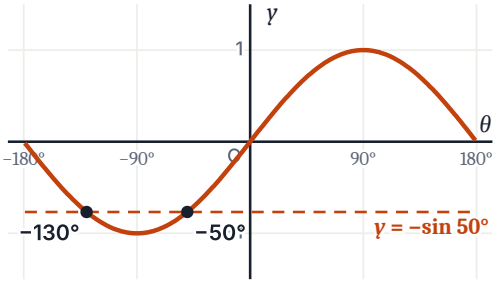
Check: the line meets the graph 4 times in $-180° \le \theta \le 540°$.

count the crossings



−115°, 65°, 245°, 425°

11 Find all the values of θ in the interval $-180° \le \theta \le 180°$ for which $\sin\theta = -\sin 50°$.



sketch the graph and the line

$\sin(-50°) = -\sin 50°$, so one value is $\theta = -50°$

sin(−θ) = −sin θ

Symmetry of the graph: $180° - (-50°) = 230°$

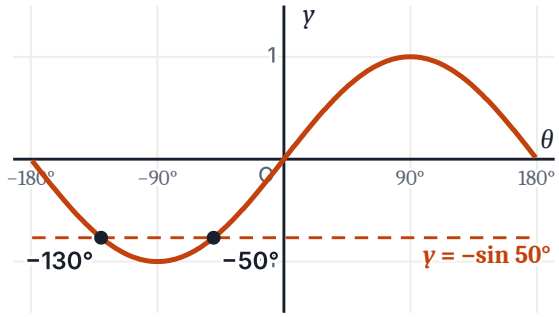
symmetry: 180° − θ

Add or subtract 360° until you leave the interval:
$\theta = -130°, -50°$

period 360°: ± 360°

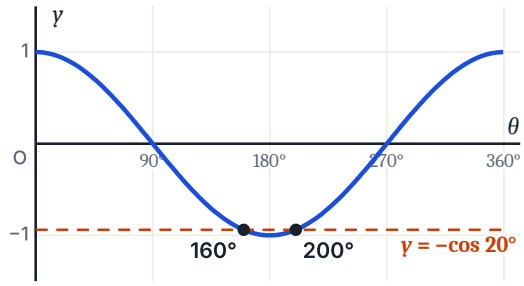
Check: the line meets the graph 2 times in $-180° \le \theta \le 180°$.

count the crossings



−130°, −50°

12 Find all the values of θ in the interval $0° \le \theta \le 360°$ for which $\cos\theta = -\cos 20°$.



sketch the graph and the line

$\cos(180° - 20°) = -\cos 20°$, so one value is $\theta = 160°$

cos(180° − θ) = −cos θ

Symmetry in the y-axis: $\cos(-160°) = \cos 160°$, so $\theta = -160°$

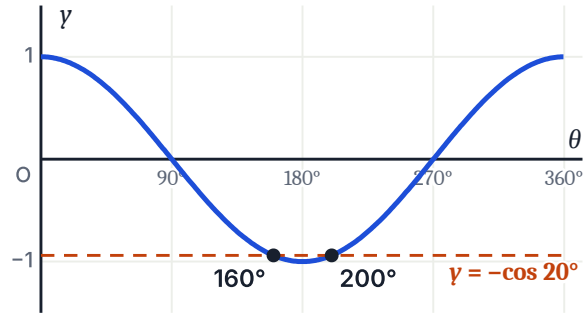
symmetry: cos(−θ) = cos θ

Add or subtract 360° until you leave the interval: $\theta = 160°, 200°$

period 360°: ± 360°

Check: the line meets the graph 2 times in $0° \le \theta \le 360°$.

count the crossings



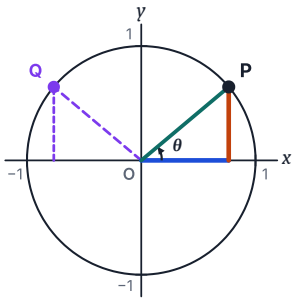
160°, 200°

D · Reasoning

13 On the unit circle, P is at angle θ and Q is at angle $180° - \theta$.

(a) Explain why Q is the reflection of P in the y-axis.

(b) Hence show that $\sin(180° - \theta) = \sin\theta$ and $\cos(180° - \theta) = -\cos\theta$.



draw the unit circle

(a) OQ is θ short of the negative x-axis (180°), just as OP is θ past the positive x-axis: mirror images in the y-axis.

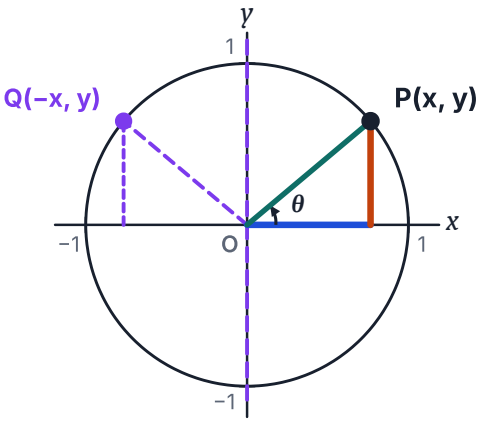
angle θ each side

(b) A reflection in the y-axis keeps y and changes the sign of x: $Q = (-\cos\theta, \sin\theta)$.

reflect: (x, y) → (−x, y)

So $\sin(180° - \theta) = \sin\theta$ and $\cos(180° - \theta) = -\cos\theta$.

read the coordinates



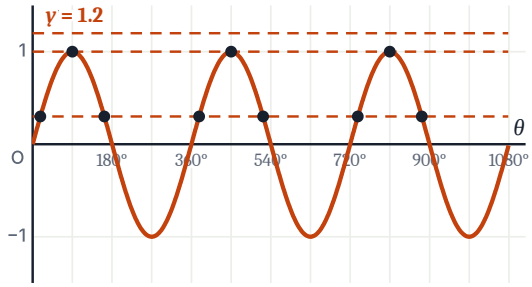
Q is P reflected in the y-axis

14 How many solutions does each equation have in the interval $0° \le \theta \le 1080°$? Explain using the graph.

(a) $\sin\theta = 0.3$

(b) $\sin\theta = 1$

(c) $\sin\theta = 1.2$



sketch the graph first

The interval is three periods of 360°. 1080° = 3 × 360°

(a) The line $y = 0.3$ crosses each wave twice: $3 \times 2 = 6$ solutions.

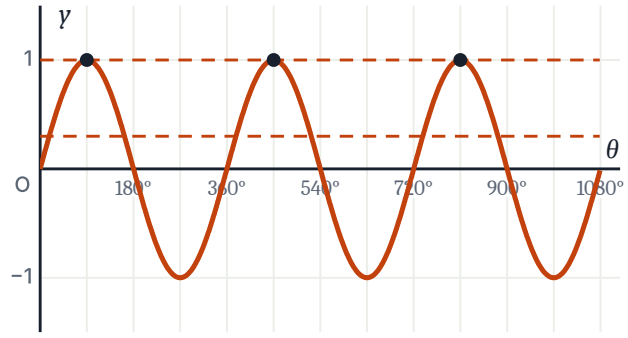
2 per period

(b) $y = 1$ only touches each maximum: 3 solutions (90°, 450°, 810°).

touches the maxima

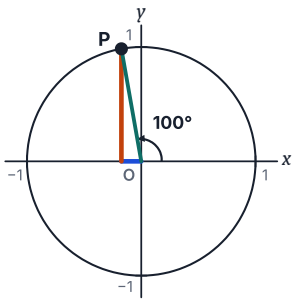
(c) $-1 \le \sin\theta \le 1$, so the line $y = 1.2$ misses the graph: no solutions.

range: none



y = 0.3: 6 crossings; y = 1: 3 touches

15 Without a calculator, put $\sin 100°$, $\cos 100°$ and $\tan 100°$ in order of size, smallest first. Explain using the unit circle.



draw the unit circle

P at 100° is just past the top of the circle: x is a small negative number and y is close to 1.

P just past (0, 1)

So $\sin 100°$ is close to 1 and $\cos 100°$ is a little below 0.

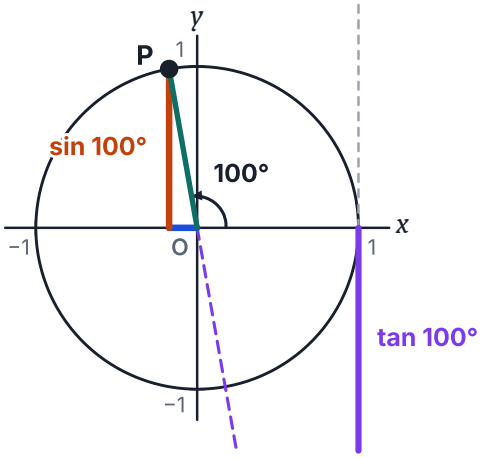
x = cos θ, y = sin θ

$\tan 100° = \dfrac{\sin 100°}{\cos 100°}$ = (close to 1) ÷ (small negative): a large negative number (OP is steep, sloping down to the right).

tan θ = y ÷ x

$\tan 100° < \cos 100° < \sin 100°$

order them

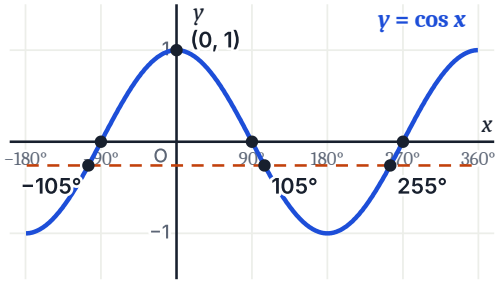


tan 100° is far below the axis

E · Exam-style question Use the mark scheme to self-assess.

16 (a) Sketch the graph of $y = \cos x$ for $-180° \le x \le 360°$. Show the coordinates of the points where the graph meets the axes. [3 marks]

(b) Hence find all the values of x in the interval $-180° \le x \le 360°$ for which $\cos x = -\cos 75°$. [3 marks]



draw the axes: mark the interval

(a) Meets the y-axis at (0, 1). **B1**

y-intercept: x = 0

Meets the x-axis at (−90°, 0), (90°, 0), (270°, 0). **B1**

zeros: 90° + 180°n

A cosine wave through these points: from (−180°, −1), up to (0, 1), down to (180°, −1) and up to (360°, 1). **B1**

correct shape

(b) $\cos(180° - 75°) = -\cos 75°$, so $x = 105°$ **M1**

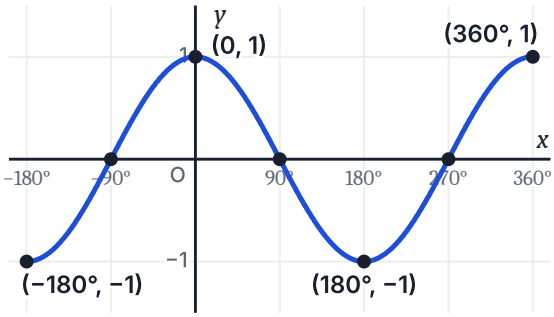
cos(180° − θ) = −cos θ

Symmetry in the y-axis: $x = -105°$ **A1**

cos(−θ) = cos θ

$-105° + 360° = 255°$. So $x = -105°, 105°, 255°$ **A1**

period 360°



(a) the sketch; (b) x = −105°, 105°, 255°