Transforming Trig Graphs

Specification: AQA 7357: E3, B9 · Edexcel 9MA0: 5.3, 2.9

INDEPENDENT **You do**

A · Key features

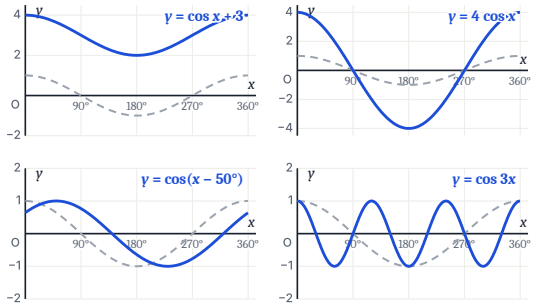
1 Describe the single transformation that maps the graph of $y = \cos x$ onto the graph of

(a) $y = \cos x + 3$

(b) $y = 4\cos x$

(c) $y = \cos(x - 50°)$

(d) $y = \cos 3x$



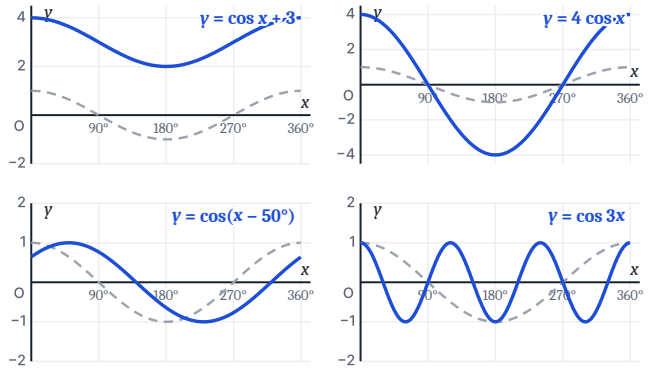
sketch each pair of graphs

(a) Translation by $\begin{pmatrix}0\\3\end{pmatrix}$: up 3 — f(x) + d: up d

(b) Stretch parallel to the y-axis, scale factor 4 — a f(x): stretch ↕ sf a

(c) Translation by $\begin{pmatrix}50°\\0\end{pmatrix}$: 50° to the right — f(x − c): right c

(d) Stretch parallel to the x-axis, scale factor ⅓ — f(bx): stretch ↔ sf 1/b



(a) up 3 (b) stretch ↕ ×4 (c) right 50° (d) stretch ↔ ×⅓

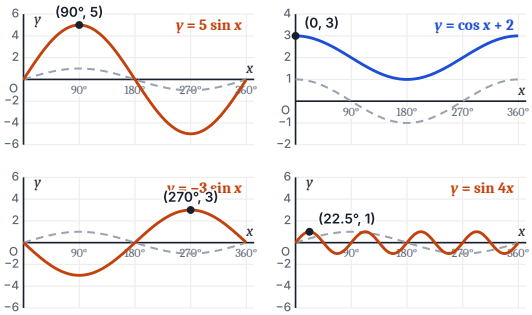
2 For each graph, $0° \le x \le 360°$, write down the maximum value, the minimum value, and the smallest value of x at which the maximum occurs.

(a) $y = 5\sin x$

(b) $y = \cos x + 2$

(c) $y = -3\sin x$

(d) $y = \sin 4x$



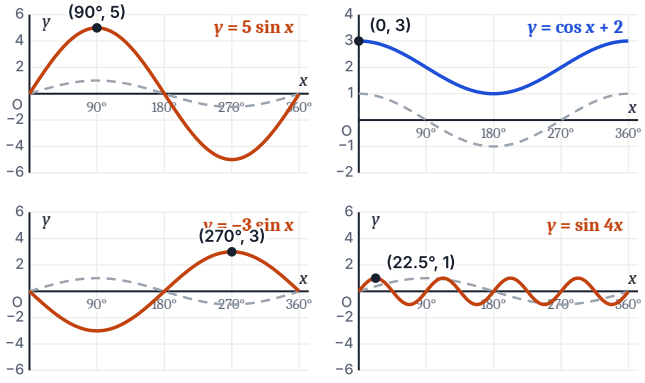
sketch each graph

(a) $y = 5\sin x$: max 5, min −5; first max at $x = 90°$ — max = a

(b) $y = \cos x + 2$: max 3, min 1; first max at $x = 0°$ — cos max at 0, then + 2

(c) $y = -3\sin x$: max 3, min −3; first max at $x = 270°$ — a < 0: max at 270°

(d) $y = \sin 4x$: max 1, min −1; first max at $x = 22.5°$ — max: 4x = 90°



max and min of each graph

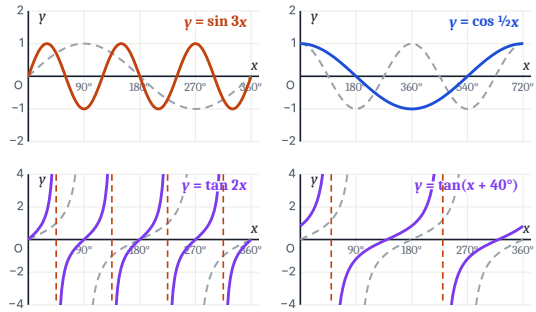
3 Write down the period of

(a) $y = \sin 3x$

(b) $y = \cos \frac{1}{2}x$

(c) $y = \tan 2x$

(d) $y = \tan(x + 40°)$



sketch each graph

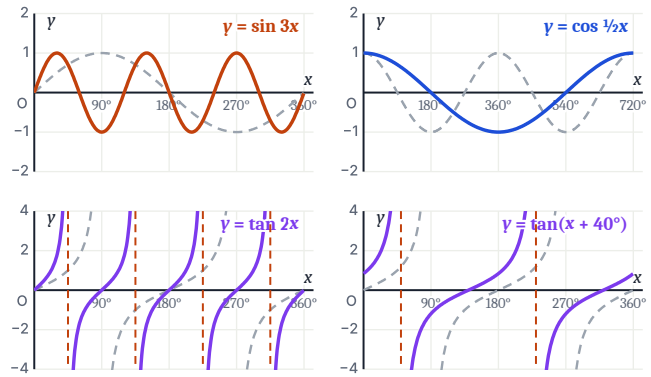
(a) $360° \div 3 = 120°$ — period 360° ÷ b

(b) $360° \div \frac{1}{2} = 720°$ — ÷ ½ = × 2

(c) $180° \div 2 = 90°$ — tan: 180° ÷ b

(d) A translation does not change the period: 180°

translation: same period

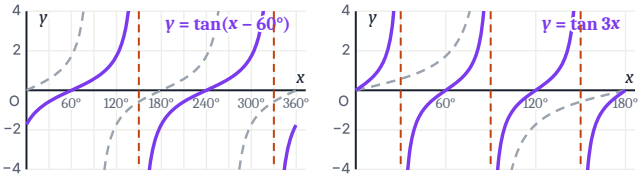


periods 120°, 720°, 90°, 180°

4 Write down the equations of the asymptotes of

(a) $y = \tan(x - 60°)$, $0° \le x \le 360°$

(b) $y = \tan 3x$, $0° \le x \le 180°$

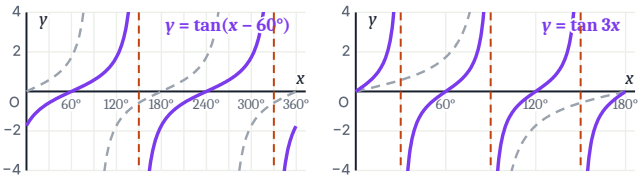


sketch each graph

(a) $x - 60° = 90°$ or $270°$: $x = 150°, 330°$ — inside = 90° + 180°n

(b) $3x = 90°, 270°, 450°$: $x = 30°, 90°, 150°$

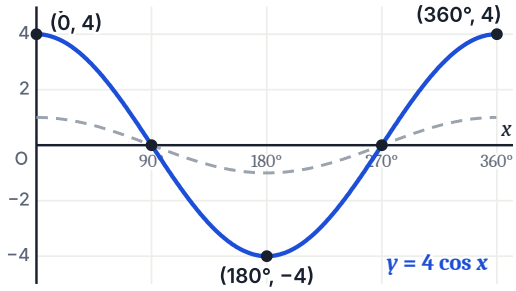
inside = 90° + 180°n



(a) x = 150°, 330° (b) x = 30°, 90°, 150°

B · Sketching

5 Sketch the graph of $y = 4\cos x$ for $0° \le x \le 360°$. Show the coordinates of the points where the graph meets the axes and of its maximum and minimum points.



draw y = f(x) dashed first

Stretch of $y = \cos x$ parallel to the y-axis, scale factor 4.

a f(x): stretch ↕ sf a

Meets the x-axis at $x = 90°, 270°$

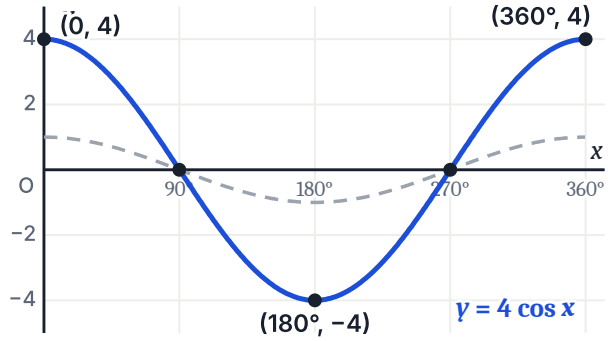
zeros: move each one

Maximum 4 at $x = 0°, 360°$; minimum -4 at $x = 180°$

max 4, min −4

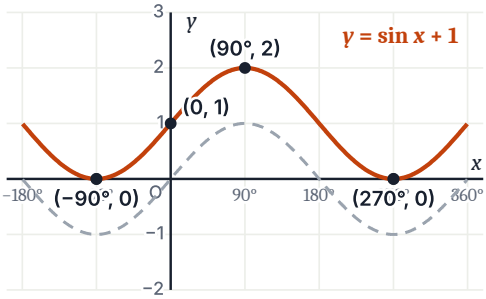
Join the points with a smooth wave. Period 360°.

smooth wave, period 360°



zeros 90°, 270°; max (0, 4), (360°, 4); min (180°, −4)

6 Sketch the graph of $y = \sin x + 1$ for $-180° \le x \le 360°$. Show the coordinates of the points where the graph meets the axes and of its maximum and minimum points.



draw y = f(x) dashed first

Translation of $y = \sin x$ by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$: 1 up.

f(x) + d: up 1

Touches the x-axis at $x = -90°, 270°$

touches at the min

Maximum 2 at $x = 90°$; minimum 0 at $x = -90°, 270°$

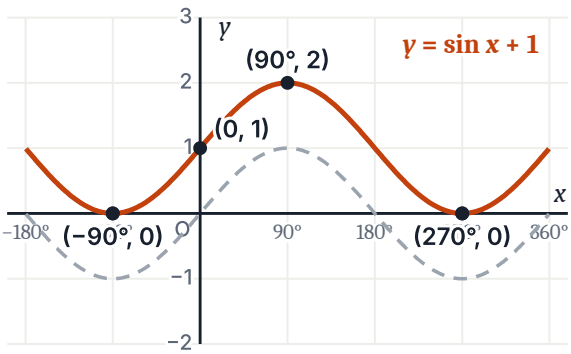
max 2, min 0

Meets the y-axis at $(0, 1)$

y-intercept: put x = 0

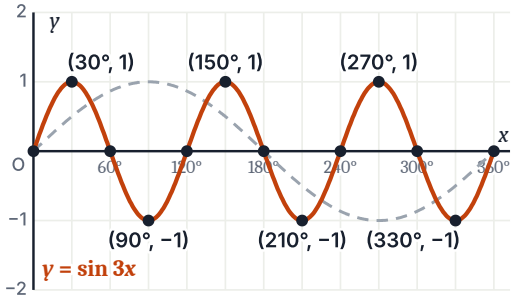
Join the points with a smooth wave. Period 360°.

smooth wave, period 360°



zeros −90°, 270°; max (90°, 2); min (−90°, 0), (270°, 0)

7 Sketch the graph of $y = \sin 3x$ for $0° \le x \le 360°$. Show the coordinates of the points where the graph meets the axes and of its maximum and minimum points.



draw y = f(x) dashed first

Stretch of $y = \sin x$ parallel to the x-axis, scale factor $\frac{1}{3}$. Period $360° \div 3 = 120°$.

f(bx): stretch ↔ sf 1/b

Meets the x-axis at $x = 0°, 60°, 120°, 180°, 240°, 300°, 360°$

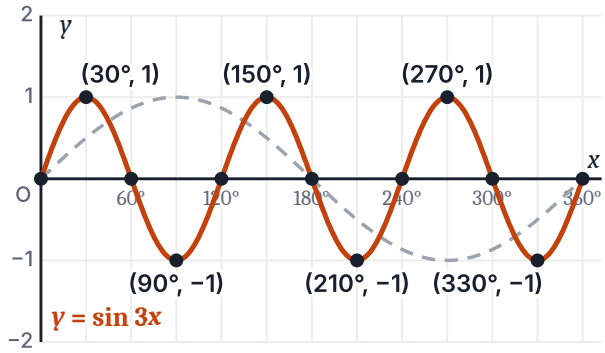
zeros: move each one

Maximum 1 at $x = 30°, 150°, 270°$; minimum −1 at $x = 90°, 210°, 330°$

max/min: move each one

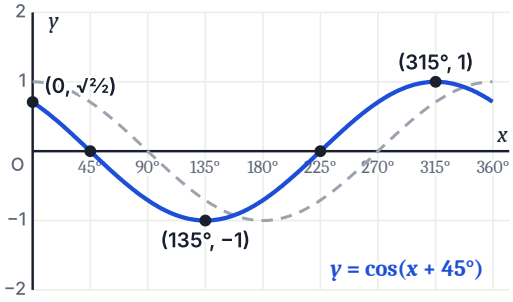
Join the points with a smooth wave. Period 120°.

period 360° ÷ 3



zeros 0°, 60°, 120°, 180°, 240°, 300°, 360°; max (30°, 1), (150°, 1), (270°, 1); min (90°, −1), (210°, −1), (330°, −1)

8 Sketch the graph of $y = \cos(x + 45°)$ for $0° \le x \le 360°$. Show the coordinates of the points where the graph meets the axes and of its maximum and minimum points.



draw y = f(x) dashed first

Translation of $y = \cos x$ by $\begin{pmatrix} -45° \\ 0 \end{pmatrix}$: 45° to the left.

f(x + c): left 45°

Meets the x-axis at $x = 45°, 225°$

zeros: move each one

Maximum 1 at $x = 315°$; minimum −1 at $x = 135°$

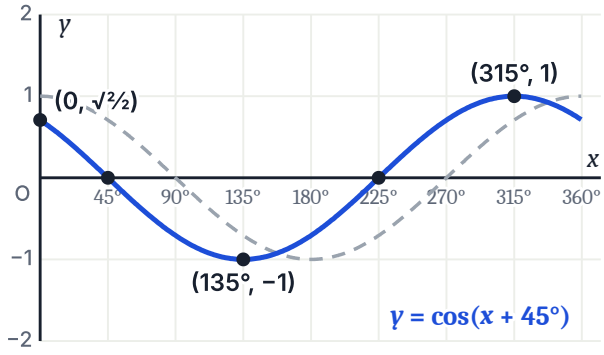
max/min: move each one

Meets the y-axis at $(0, \frac{\sqrt{2}}{2})$

y-intercept: put x = 0

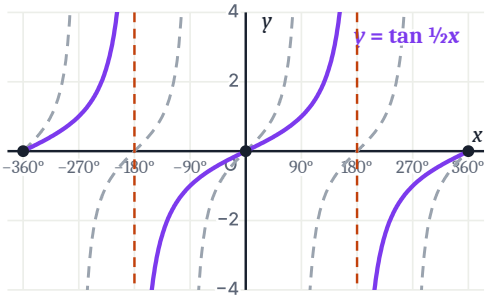
Join the points with a smooth wave. Period 360°.

smooth wave, period 360°



zeros 45°, 225°; max (315°, 1); min (135°, −1)

9 Sketch the graph of $y = \tan \frac{1}{2}x$ for $-360° \le x \le 360°$. Show the asymptotes and the coordinates of the points where the graph meets the axes.



draw y = f(x) dashed first

Stretch of $y = \tan x$ parallel to the x-axis, scale factor 2.
Period $180° \div \frac{1}{2} = 360°$.

f(bx): stretch ↔ sf 1/b

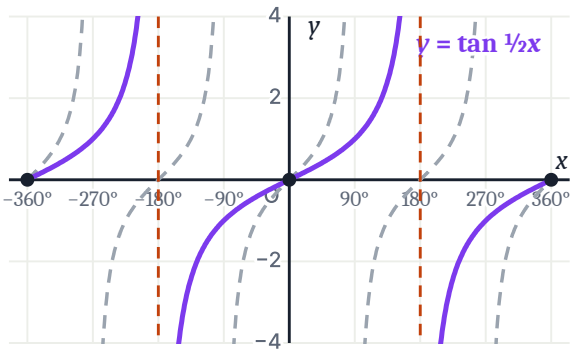
Asymptotes (where tan is undefined): $x = -180°, 180°$

move the asymptotes too

Meets the x-axis at $x = -360°, 0°, 360°$ zeros: move each one

Each branch rises between two asymptotes, through a zero. Period 360°.

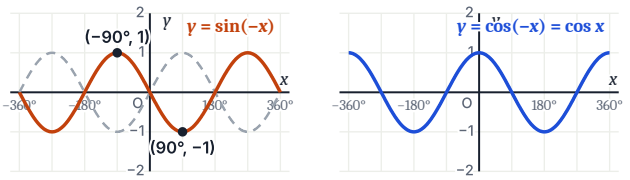
period 180° ÷ ½



asymptotes x = −180°, 180°; zeros −360°, 0°, 360°

C · Reasoning No calculator for this section.

10 (a) Describe the transformation that maps the graph of $y = \sin x$ onto the graph of $y = \sin(-x)$.
(b) Use graphs to explain why $\sin(-x) = -\sin x$.
(c) Explain why $\cos(-x) = \cos x$.



sketch both graphs

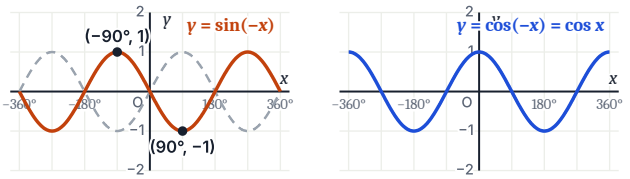
(a) Reflection in the y-axis. f(−x): reflect in the y-axis

(b) Reflecting $y = \sin x$ in the y-axis sends (90°, 1) to (−90°, 1) and (−90°, −1) to (90°, −1): the same graph as reflecting it in the x-axis, $y = -\sin x$.

same graph as −sin x

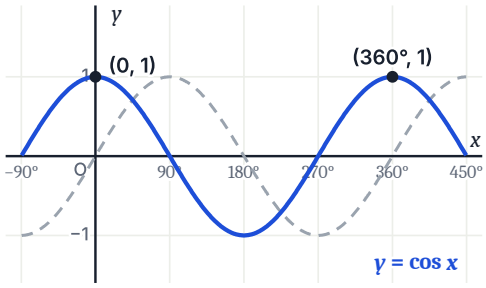
(c) The graph of $y = \cos x$ is symmetrical about the y-axis, so reflecting it in the y-axis changes nothing.

cos: symmetrical in the y-axis



sin(−x) = −sin x; cos(−x) = cos x

11 The graph of $y = \sin x$ can be translated onto the graph of $y = \cos x$.
(a) Write down a translation vector that does this.
(b) Write down a different translation vector that also does this.
(c) Hence write $\cos x$ in the form $\sin(x + c)$.



sketch both graphs

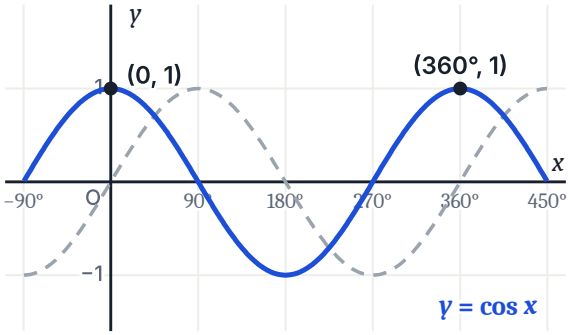
(a) The maximum of $\sin x$ is at 90°; the maximum of $\cos x$ is at 0. Move 90° left: $\begin{pmatrix} -90° \\ 0 \end{pmatrix}$

match a max point

(b) Or move 270° right, to the next maximum at 360°: $\begin{pmatrix} 270° \\ 0 \end{pmatrix}$

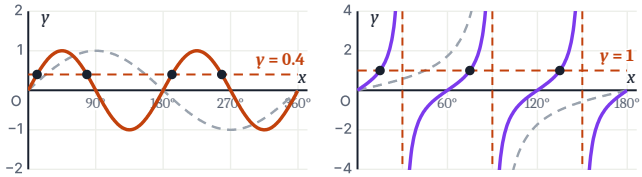
± one period

(c) $\cos x = \sin(x + 90°)$ f(x + c): c to the left



cos x = sin(x + 90°): max 90° → 0

12 Use sketches to find the number of solutions of
(a) $\sin 2x = 0.4$, $0° \le x \le 360°$
(b) $\tan 3x = 1$, $0° \le x \le 180°$.



sketch both graphs

(a) $y = \sin 2x$ has period 180°: two waves in 360°.

period 360° ÷ 2

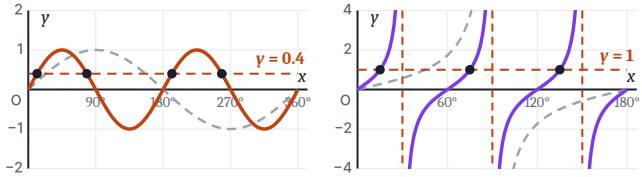
The line $y = 0.4$ crosses each wave twice: 4 solutions.

2 per wave

(b) $y = \tan 3x$ has period 60°; asymptotes at 30°, 90°, 150°: three branches cross $y = 1$ (one more would start at 180°, outside).

period 180° ÷ 3
one per branch

3 solutions.

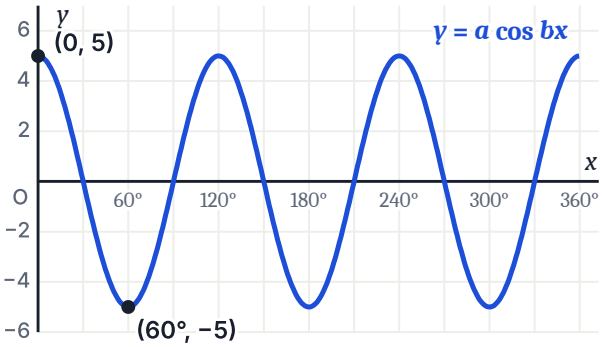


(a) 4 crossings (b) 3 crossings

D · Finding constants · combined transformations

Question 15 combines transformations: A-level content for AQA (extension at AS).

13 The graph shows $y = a\cos bx$, $0° \le x \le 360°$, where a and b are positive. It passes through (0, 5) and its first minimum point is (60°, −5). Find a and b.



The maximum value is 5, so $a = 5$

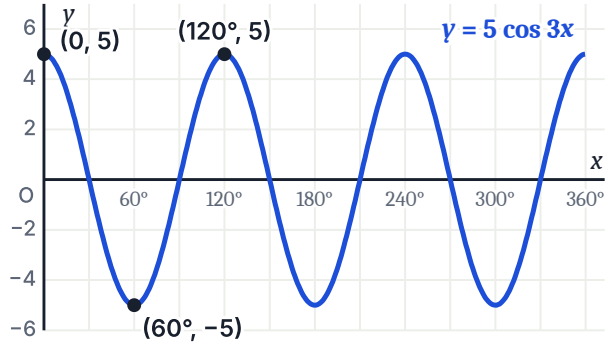
max → a

From the maximum at 0 to the minimum at 60° is half a wave: period 120°.

max to min: half a period

$360° \div b = 120°$, so $b = 3$. Check: $5\cos(3 \times 60°) = 5\cos 180° = -5$ ✓

period 360° ÷ b

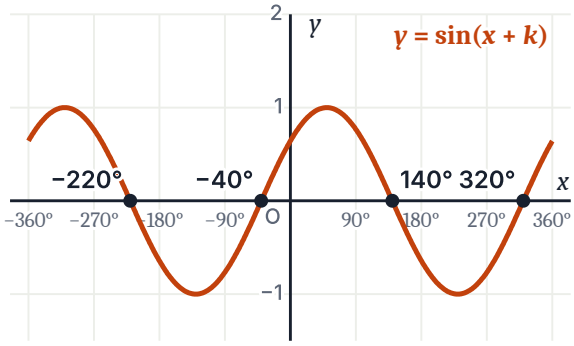


a = 5, b = 3: period 120°

14 The graph shows $y = \sin(x + k)$, $-360° \le x \le 360°$. It meets the x-axis at $-220°$, $-40°$, $140°$ and $320°$, and is increasing at $-40°$.

(a) Find k, given that $0 < k < 360°$.

(b) Give a different value of k that gives the same graph.

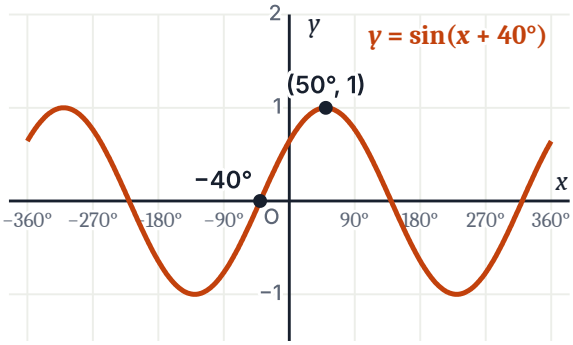


(a) sin crosses zero going up at 0°, so $-40° + k = 0$.
$k = 40°$. Check: $140° + 40° = 180°$, a zero ✓

sin: rising zero at 0°
solve x + k = 0

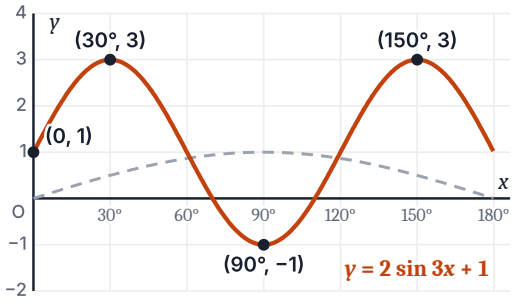
(b) Adding a period gives the same graph: $k = 40° + 360° = 400°$ (or $40° - 360° = -320°$).

± one period



k = 40°: the graph of sin x moved 40° left

15 *Extension: combined transformations.* For the graph of $y = 2\sin 3x + 1$, $0° \le x \le 180°$:
(a) describe, in order, the three transformations that map $y = \sin x$ onto it
(b) sketch it, showing the coordinates of its maximum and minimum points and of the point where it meets the y-axis.



draw y = f(x) dashed first

(a) Stretch of $y = \sin x$ parallel to the x-axis, scale factor $\frac{1}{3}$. Period $360° \div 3 = 120°$.

f(bx): stretch ↔ sf 1/b

Then a stretch parallel to the y-axis, scale factor 2.

a f(x): stretch ↕ sf a

Then a translation by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$: 1 up.

f(x) + d: up 1

(b) Maximum 3 at $x = 30°, 150°$; minimum -1 at $x = 90°$

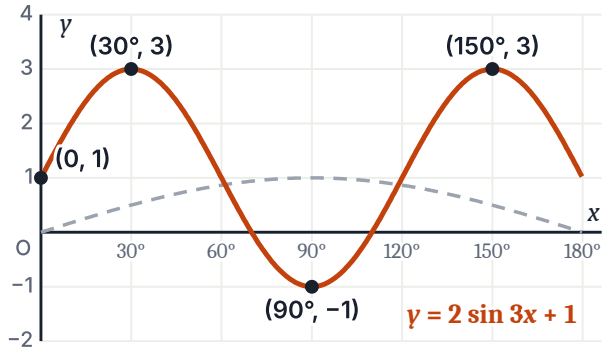
max 3, min −1

Meets the y-axis at $(0, 1)$

y-intercept: put x = 0

Join the points with a smooth wave. Period 120°.

period 360° ÷ 3



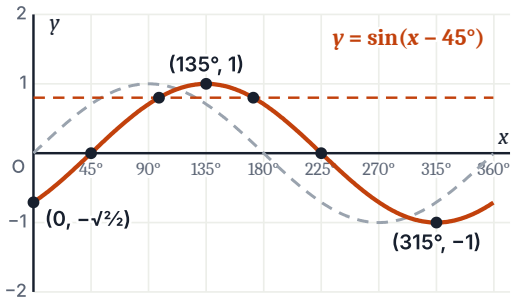
max (30°, 3), (150°, 3); min (90°, −1)

E · Exam-style question Use the mark scheme to self-assess.

16 (a) Sketch the graph of $y = \sin(x - 45°)$ for $0° \le x \le 360°$. Show the coordinates of the points where the graph meets the axes. [3 marks]

(b) Write down the coordinates of the maximum and minimum points. [2 marks]

(c) State the number of solutions of $\sin(x - 45°) = 0.8$ for $0° \le x \le 360°$. [1 mark]



draw the axes: mark the interval

(a) $x = 0$: $y = \sin(-45°) = -\frac{\sqrt{2}}{2}$. Meets the y-axis at $(0, -\frac{\sqrt{2}}{2})$ **B1**

y-intercept: put x = 0

Zeros where $x - 45° = 0°, 180°$: (45°, 0), (225°, 0) **B1**

f(x − 45°): right 45°

A sine wave moved 45° right, through these points. **B1**

correct shape

(b) Maximum where $x - 45° = 90°$: (135°, 1) **B1**

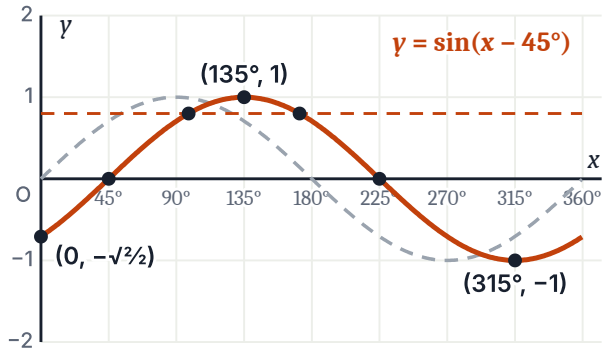
max: inside = 90°

Minimum where $x - 45° = 270°$: (315°, −1) **B1**

min: inside = 270°

(c) The line $y = 0.8$ crosses the wave twice: 2 solutions. **B1**

count the crossings



max (135°, 1), min (315°, −1); y = 0.8 twice