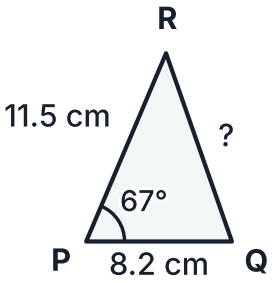
Unit Review

Specification: AQA 7357: E1, E3, B9 · Edexcel 9MA0: 5.1, 5.3, 2.9

INDEPENDENT **Mixed practice**

A · Fluency

1 In triangle PQR, $PQ = 8.2$ cm, $PR = 11.5$ cm and angle $QPR = 67°$. Find QR.



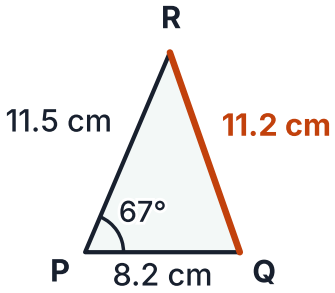
draw a diagram first

$QR^2 = 8.2^2 + 11.5^2 - 2(8.2)(11.5)\cos 67° = 125.79...$ from L1

cosine rule

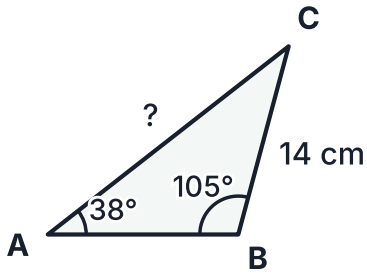
$QR = 11.215...$, so $QR = 11.2$ cm

square root; 3 s.f.



QR = 11.2 cm

2 In triangle ABC, angle $BAC = 38°$, angle $ABC = 105°$ and $BC = 14$ cm. Find AC.



draw a diagram first

AC is opposite B; BC is opposite A. from L2

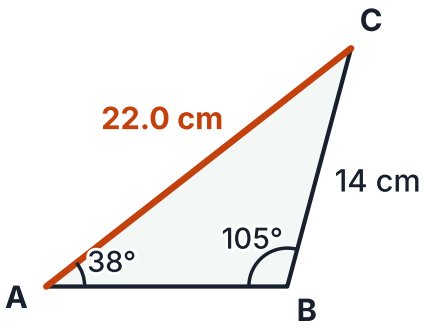
label: side opposite angle

$$\frac{AC}{\sin 105°} = \frac{14}{\sin 38°}$$

sine rule

$$AC = \frac{14 \sin 105°}{\sin 38°} = 21.964..., \text{ so } AC = 22.0 \text{ cm}$$

rearrange; 3 s.f.



AC = 22.0 cm

3 A triangle has sides 13 cm and 9 cm with an angle of 128° between them. Find its area.



draw a diagram first

Area = ½ × 13 × 9 × sin 128° from L4

½ab sin C

= 46.098..., so 46.1 cm²

calculator, 3 s.f.



Area = 46.1 cm²

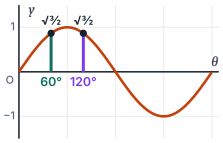
area 46.1 cm²

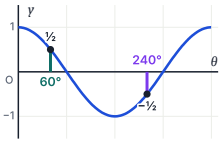
4 No calculator. Given that $\sin 60° = \frac{\sqrt{3}}{2}$ and $\cos 60° = \frac{1}{2}$, use the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ to find the exact value of

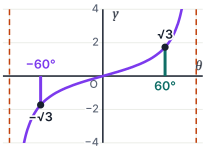
(a) $\sin 120°$

(b) $\cos 240°$

(c) $\tan(-60°)$







sketch the graphs: mark the angles

(a) Symmetry of $y = \sin x$:

$\sin 120° = \sin(180° - 60°) = \sin 60° = \frac{\sqrt{3}}{2}$ from L6

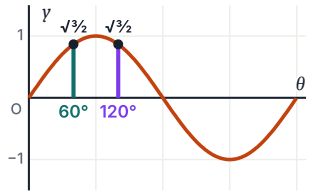
symmetry: 180° − θ

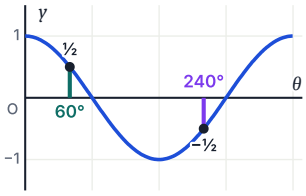
(b) $\cos 240° = \cos(360° - 120°) = \cos 120°$, and $\cos 120° = -\cos 60° = -\frac{1}{2}$

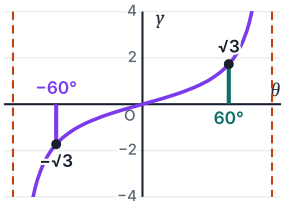
cos(180° − θ) = −cos θ

(c) $\tan 60° = \frac{\sqrt{3}}{2} \div \frac{1}{2} = \sqrt{3}$, and $\tan(-60°) = -\tan 60° = -\sqrt{3}$

tan(−θ) = −tan θ

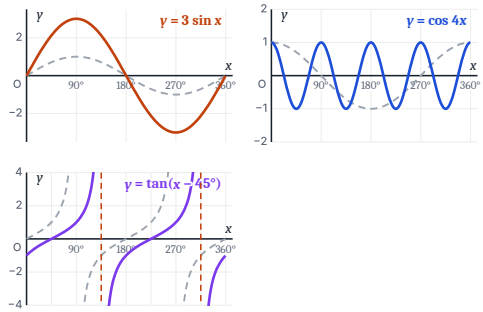






sin 120° = √3/2; cos 240° = −½; tan(−60°) = −√3

5 Describe the single transformation that maps
(a) $y = \sin x$ onto $y = 3\sin x$
(b) $y = \cos x$ onto $y = \cos 4x$
(c) $y = \tan x$ onto $y = \tan(x - 45°)$
and write down the period of each new graph.



sketch each pair of graphs

(a) Stretch parallel to the y-axis, scale factor 3. Period 360°. from L7

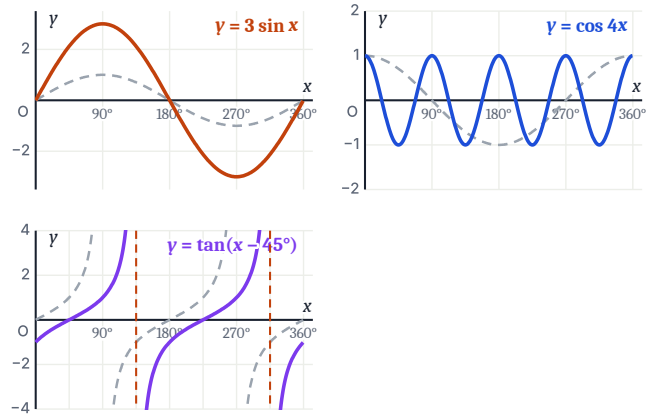
a f(x): stretch ↕ sf a

(b) Stretch parallel to the x-axis, scale factor ¼. Period $360° \div 4 = 90°$.

f(bx): stretch ↔ sf 1/b

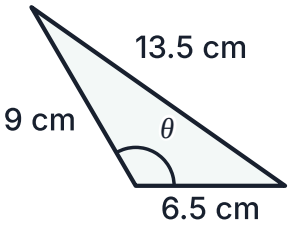
(c) Translation by $\begin{pmatrix} 45° \\ 0 \end{pmatrix}$: 45° to the right. Period 180° (a translation does not change it).

f(x − c): right c



(a) stretch ↕ ×3 (b) stretch ↔ ×¼ (c) right 45°

6 A triangle has sides 6.5 cm, 9 cm and 13.5 cm. Find the size of its largest angle.



draw a diagram first

The largest angle θ is opposite the longest side, 13.5 cm. from L1

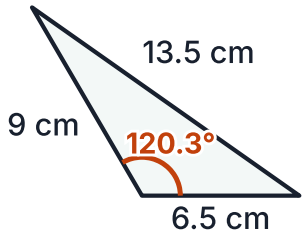
largest angle: longest side

$$\cos\theta = \frac{6.5^2 + 9^2 - 13.5^2}{2 \times 6.5 \times 9} = \frac{-59}{117} = -0.5042...$$

rearrange for cos A

$\theta = \cos^{-1}(-0.5042...) = 120.28...°$, so $\theta = 120.3°$

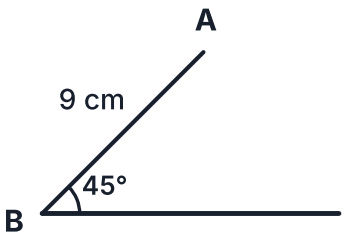
$\cos^{-1}$ (negative → obtuse)



largest angle 120.3°

B · Multi-step triangles

7 In triangle ABC, $AB = 9$ cm, $AC = 7$ cm and angle $ABC = 45°$. Find the two possible lengths of BC.



AC = 7 cm: where is C?

draw a diagram first

$\frac{\sin C}{9} = \frac{\sin 45°}{7} \Rightarrow \sin C = 0.90913...$ from L3

sine rule: C opposite AB

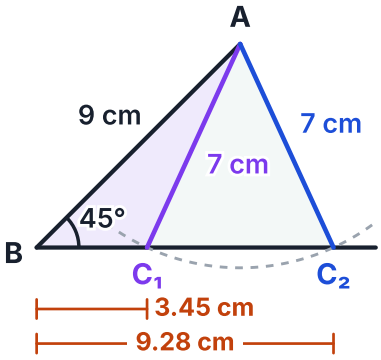
$C = 65.386...°$ or $180° - 65.386...° = 114.61...°$;
$45° + 114.6° < 180°$: both fit.

180° − θ: does it fit?

$A = 180° - 45° - C = 69.613...°$ or $20.386...°$ angle sum

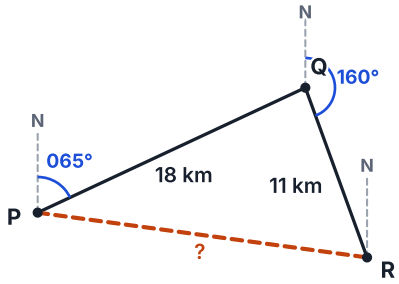
$BC = \frac{7 \sin A}{\sin 45°}$: $BC = 9.28$ cm or 3.45 cm

sine rule again: two answers



BC = 9.28 cm or 3.45 cm

8 A ship sails 18 km from P to Q on a bearing of 065°, then 11 km from Q to R on a bearing of 160°. Find
(a) the distance PR
(b) the bearing of R from P.



draw a diagram: North lines first

(a) Back bearing from Q to P: $065° + 180° = 245°$. Angle $PQR = 245° - 160° = 85°$ from L5, L1

bearings: angle between

$PR^2 = 18^2 + 11^2 - 2(18)(11)\cos 85° = 410.48...$ cosine rule

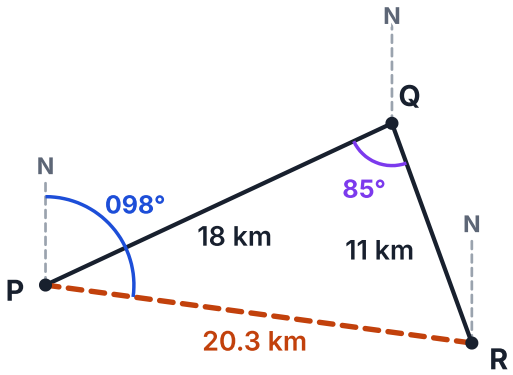
$PR = 20.260...$, so $PR = 20.3$ km square root; 3 s.f.

(b) $\sin QPR = \frac{11 \sin 85°}{20.260...} \Rightarrow QPR = 32.742...°$

sine rule: the acute angle

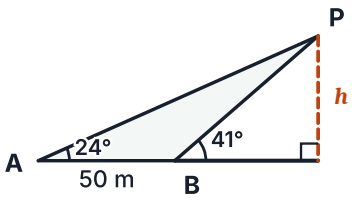
Bearing $065° + 32.742...° = 97.742...°$, so 098°

add to the first bearing



PR = 20.3 km on 098°

9 From a point A, the angle of elevation of the top T of a tower is 24°. From a point B, 50 m nearer the foot of the tower on the same level ground, the angle of elevation is 41°. Find the height of the tower.



draw a diagram first

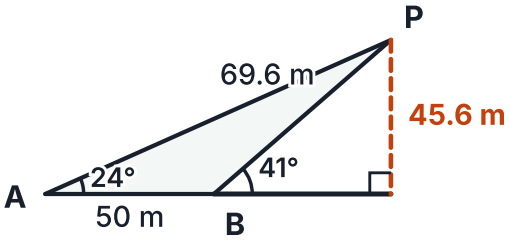
In triangle ABT: angle $ATB = 41° - 24° = 17°$ and angle $ABT = 180° - 41° = 139°$. from L5, L2

exterior angle: 41° − 24°

$\frac{BT}{\sin 24°} = \frac{50}{\sin 17°} \Rightarrow BT = 69.558...$ sine rule: BT opposite A

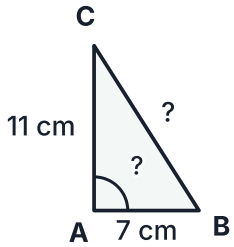
Height $= BT \sin 41° = 45.634...$, so 45.6 m

right angle: opp = hyp × sin



height 45.6 m

10 In triangle ABC, $AB = 7$ cm, $AC = 11$ cm and the area is 30 cm². Angle BAC is obtuse. Find BC.



draw a diagram first

area 30 cm²; angle A obtuse

$\frac{1}{2} \times 7 \times 11 \times \sin A = 30 \Rightarrow \sin A = \frac{60}{77} = 0.77922...$ from L4, L1

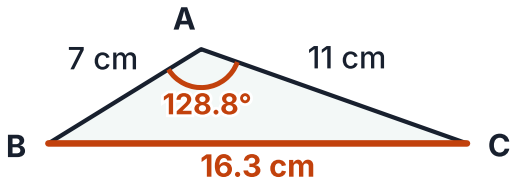
½ab sin C: rearrange

$A = 51.189...°$ (acute) or $180° - 51.189...° = 128.81...°$.
Obtuse, so $A = 128.81...°$

180° − θ: the obtuse one

$BC^2 = 7^2 + 11^2 - 2(7)(11)\cos 128.81...° = 266.51...$ cosine rule

$BC = 16.325...$, so $BC = 16.3$ cm square root; 3 s.f.



Area = 30 cm²

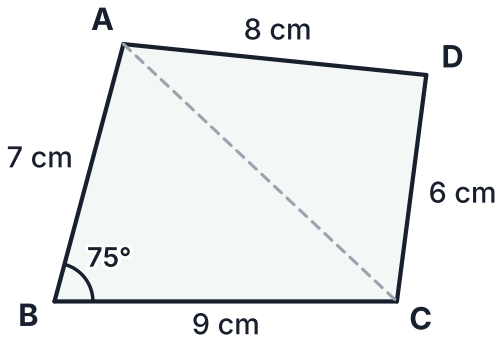
A = 128.8°, BC = 16.3 cm

11 The diagram shows a quadrilateral $ABCD$ with $AB = 7$ cm, $BC = 9$ cm, $CD = 6$ cm, $DA = 8$ cm and angle $ABC = 75°$. Find

(a) the length of the diagonal AC

(b) angle ADC

(c) the area of $ABCD$.



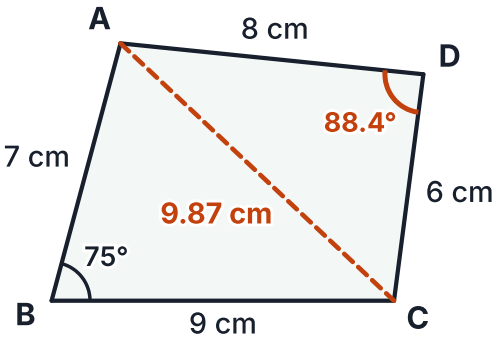
(a) $AC^2 = 7^2 + 9^2 - 2(7)(9)\cos 75° = 97.388...$, so $AC = 9.8685... = 9.87$ cm from L5, L1, L4 — cosine rule in ABC

(b) $\cos D = \dfrac{8^2 + 6^2 - AC^2}{2 \times 8 \times 6} = 0.02719...$ — cosine rule: full AC

$ADC = 88.441...°$, so $88.4°$ — $\cos^{-1}$; 1 d.p.

(c) Area $= \frac{1}{2}(7)(9)\sin 75° + \frac{1}{2}(8)(6)\sin 88.441...°$ — split by the diagonal

$= 30.426... + 23.991... = 54.417...$, so 54.4 cm² — add the two areas

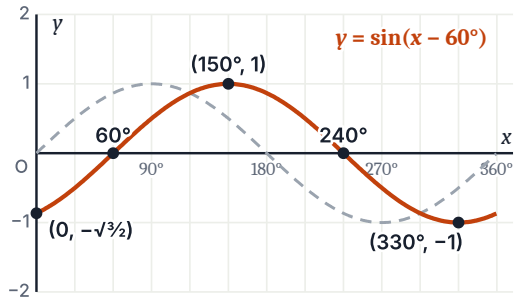


AC = 9.87 cm, ADC = 88.4°

area 54.4 cm²

C · Graphs and transformations No calculator for this section.

12 Sketch the graph of $y = \sin(x - 60°)$ for $0° \le x \le 360°$. Show the coordinates of the points where the graph meets the axes and of its maximum and minimum points.



draw y = f(x) dashed first

Translation of $y = \sin x$ by $\begin{pmatrix} 60° \\ 0 \end{pmatrix}$: 60° to the right. from L7

f(x + c): right 60°

Meets the x-axis at $x = 60°, 240°$ zeros: move each one

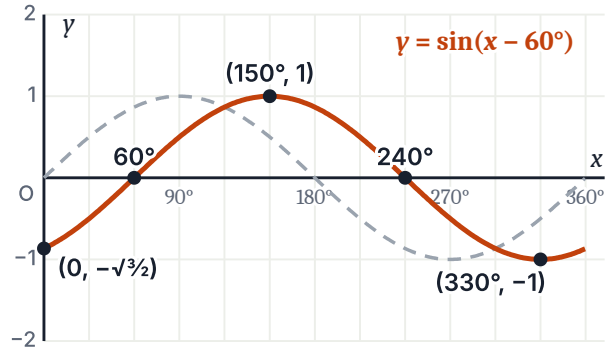
Maximum 1 at $x = 150°$; minimum -1 at $x = 330°$

max/min: move each one

Meets the y-axis at $(0, -\frac{\sqrt{3}}{2})$ y-intercept: put x = 0

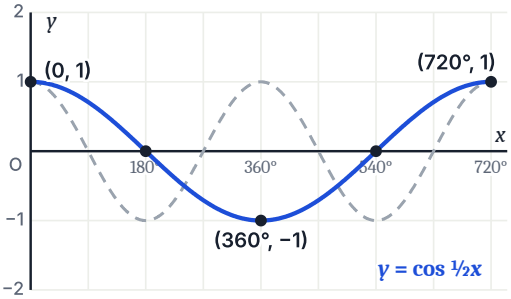
Join the points with a smooth wave. Period 360°.

smooth wave, period 360°



zeros 60°, 240°; max (150°, 1); min (330°, −1)

13 Sketch the graph of $y = \cos ½x$ for $0° \le x \le 720°$. Show the coordinates of the points where the graph meets the axes and of its maximum and minimum points.



draw y = f(x) dashed first

Stretch of $y = \cos x$ parallel to the x-axis, scale factor 2. Period $360° \div ½ = 720°$. from L7

f(bx): stretch ↔ sf 1/b

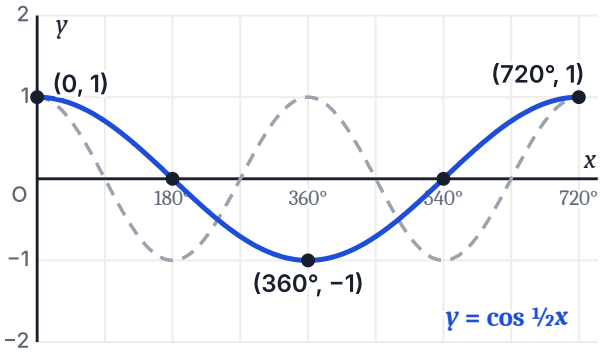
Meets the x-axis at $x = 180°, 540°$ zeros: move each one

Maximum 1 at $x = 0°, 720°$; minimum -1 at $x = 360°$

max/min: move each one

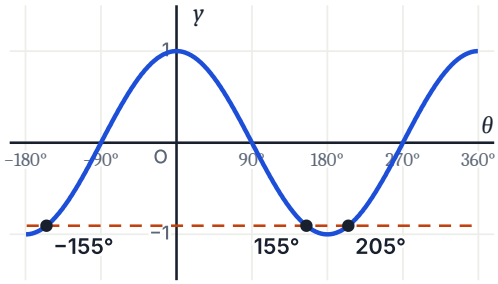
Join the points with a smooth wave. Period 720°.

period 360° ÷ ½



zeros 180°, 540°; max (0, 1), (720°, 1); min (360°, −1)

14 Use the graph of $y = \cos\theta$ to find all the values of θ in the interval $-180° \le \theta \le 360°$ for which $\cos\theta = -\cos 25°$.



sketch the graph and the line

$\cos(180° - 25°) = -\cos 25°$, so one value is $\theta = 155°$ from L6

cos(180° − θ) = −cos θ

Symmetry in the y-axis: $\cos(-155°) = \cos 155°$, so $\theta = -155°$

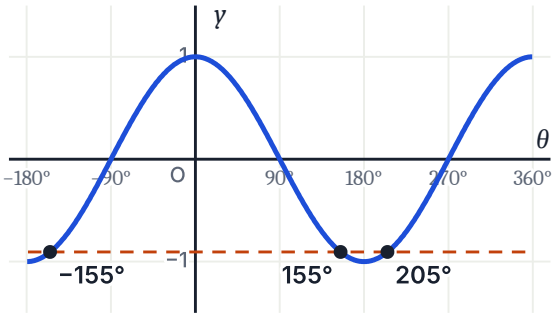
symmetry: cos(−θ) = cos θ

Add or subtract 360° until you leave the interval: $\theta = -155°, 155°, 205°$

period 360°: ± 360°

Check: the line meets the graph 3 times in $-180° \le \theta \le 360°$.

count the crossings

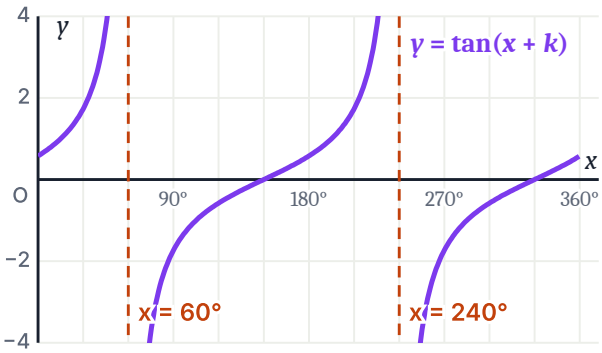


θ = −155°, 155°, 205°

15 The graph shows $y = \tan(x + k)$, $0° \le x \le 360°$, where k is a constant with $0 < k < 90°$. Its asymptotes are $x = 60°$ and $x = 240°$.

(a) Find k.

(b) Find the exact coordinates of the point where the graph meets the y-axis.



(a) The first asymptote of $y = \tan x$ is at 90°. The graph is moved left, so $60° + k = 90°$ from L7, L6

asymptote: inside = 90°

solve x + k = 90°

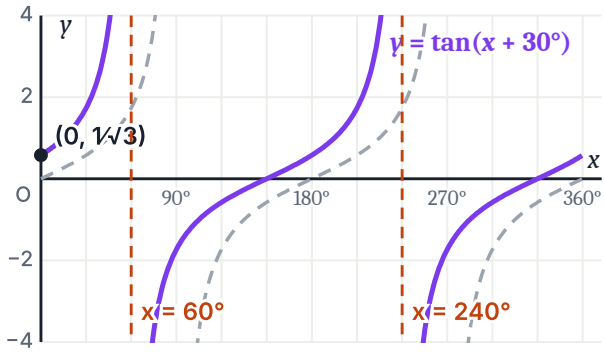
$k = 30°$. Check: $240° + 30° = 270°$, an asymptote of tan ✓

(b) $x = 0$: $y = \tan 30° = \dfrac{\sin 30°}{\cos 30°} = ½ \div \dfrac{\sqrt{3}}{2} = \dfrac{1}{\sqrt{3}}$

y-intercept: put x = 0

The point is $\left(0, \dfrac{1}{\sqrt{3}}\right)$

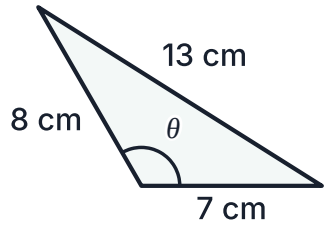
exact: leave the root



k = 30°: tan x moved 30° left

D · Reasoning, exact values and "show that" No calculator for this section.

16 A triangle has sides 7 cm, 8 cm and 13 cm.
(a) Show that its largest angle is 120°.
(b) Find the exact area of the triangle.



draw a diagram first

(a) The largest angle θ is opposite 13 cm:

$$\cos\theta = \frac{7^2 + 8^2 - 13^2}{2 \times 7 \times 8} = \frac{49 + 64 - 169}{112}$$ from L1, L4

largest angle: longest side

$\cos\theta = \frac{-56}{112} = -\frac{1}{2}$, and $\cos 120° = -\cos 60° = -\frac{1}{2}$, so $\theta = 120°$ ✓

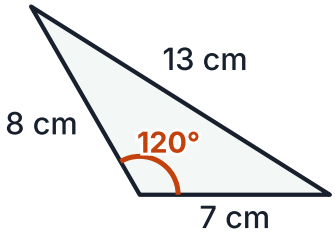
cos(180° − θ) = −cos θ

(b) Area $= \frac{1}{2} \times 7 \times 8 \times \sin 120° = 28 \times \frac{\sqrt{3}}{2}$

½ab sin C; sin 120° = sin 60°

$= 14\sqrt{3}$ cm²

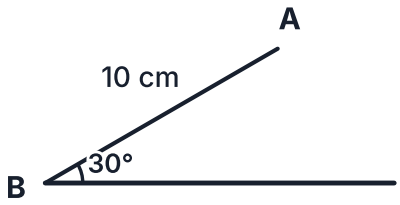
exact: leave the root



Area = 14√3 cm²

θ = 120°; area 14√3 cm²

17 In triangle *ABC*, *AB* = 10 cm and angle *ABC* = 30°. The side *AC* has length *b* cm. Find the values of *b* for which there are
(a) two possible triangles
(b) exactly one triangle
(c) no triangle.



AC = b cm: where is C?

draw a diagram first

The shortest distance from A to the line BC is $h = 10 \sin 30° = 5$. from L3

shortest distance: c sin B

(a) The arc of radius b from A meets the line twice, both on the same side of B, when $5 < b < 10$

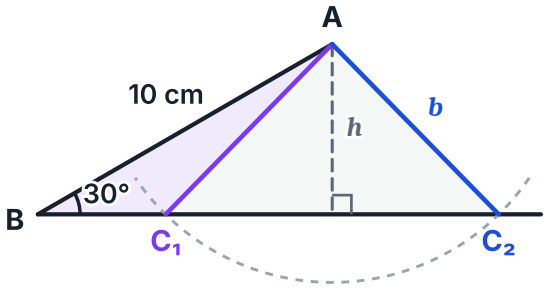
two crossings: h < b < c

(b) Exactly one when $b = 5$ (a right angle at C) or $b \geq 10$ (the second crossing is behind B)

one: b = h or b ≥ c

(c) None when $b < 5$: the side cannot reach the line.

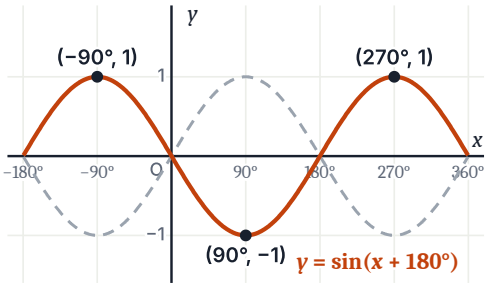
none: b < h



h = 5: two triangles when 5 < b < 10

h = 5; two triangles when 5 < b < 10

18 (a) Describe the single transformation that maps the graph of $y = \sin x$ onto the graph of $y = \sin(x + 180°)$.
(b) Use graphs to show that $\sin(x + 180°) = -\sin x$.
(c) Hence find the exact value of $\sin 240°$.



draw y = f(x) dashed first

(a) Translation by $\begin{pmatrix} -180° \\ 0 \end{pmatrix}$: 180° to the left. from L6, L7

f(x + c): left c

(b) The maximum (90°, 1) moves to (−90°, 1); the minimum (270°, −1) moves to (90°, −1).

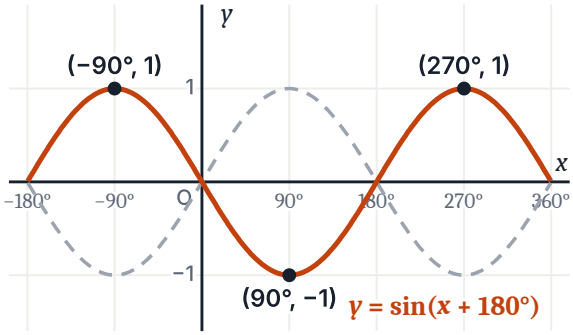
move the key points

That is $y = \sin x$ reflected in the x-axis: the graph of $y = -\sin x$. So $\sin(x + 180°) = -\sin x$ ✓

same graph as −sin x

(c) $\sin 240° = \sin(60° + 180°) = -\sin 60° = -\dfrac{\sqrt{3}}{2}$

use x = 60°



sin(x + 180°) = −sin x

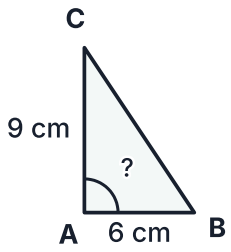
E · Exam-style questions Use the mark scheme to self-assess. Each question uses more than one part of the unit.

19 *AQA-style*. In triangle ABC, $AB = 6$ cm, $AC = 9$ cm and the area of the triangle is 18 cm².

(a) Show that $\sin BAC = \frac{2}{3}$. [2 marks]

(b) Explain, using the graph of $y = \sin x$, why angle BAC has two possible values. [1 mark]

(c) Given that angle BAC is obtuse, find the length of BC. [4 marks]



draw a diagram first

area 18 cm²

(a) $\frac{1}{2} \times 6 \times 9 \times \sin BAC = 18$ from L4, L6, L1 **M1** — ½ab sin C

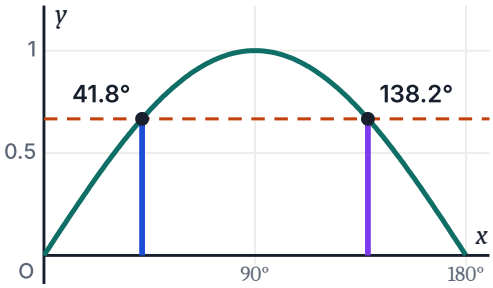
$27 \sin BAC = 18$, so $\sin BAC = \frac{2}{3}$ ✓ **A1** — ÷ 27

(b) $y = \frac{2}{3}$ meets $y = \sin x$ twice in $0° < x < 180°$: at θ and $180° - \theta$ **B1** — symmetry: 180° − θ

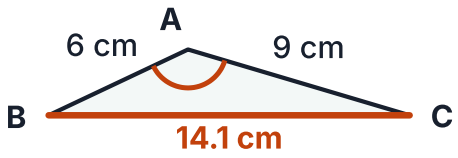
(c) Obtuse: $180° - \sin^{-1}(\frac{2}{3}) = 138.18...°$ **B1** — 180° − θ: the obtuse one

$BC^2 = 6^2 + 9^2 - 2(6)(9)\cos 138.18...° = 197.49...$ **M1 A1** — cosine rule

$BC = 14.053...$, so $BC = 14.1$ cm **A1** — square root; 3 s.f.



sin x = ⅔ twice

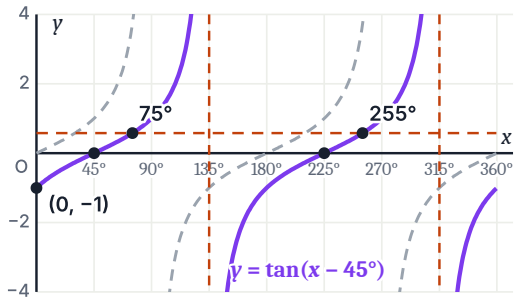


A = 138.2°; BC = 14.1 cm

A = 138.2° (obtuse), BC = 14.1 cm

20 *Edexcel-style*. (a) Sketch the graph of $y = \tan(x - 45°)$ for $0° \le x \le 360°$. Show the asymptotes and the coordinates of the points where the graph meets the axes. [3 marks]

(b) Given that $\tan 30° = \frac{1}{\sqrt{3}}$, use your graph to solve $\tan(x - 45°) = \frac{1}{\sqrt{3}}$ for $0° \le x \le 360°$. [3 marks]



draw the axes: mark the interval

(a) Asymptotes where $x - 45° = 90°, 270°$: $x = 135°$, $x = 315°$ from L7, L6 **B1**

move the asymptotes too

Meets the x-axis at (45°, 0), (225°, 0); meets the y-axis at (0, −1), as $\tan(-45°) = -1$ **B1**

zeros: right 45°

Branches rising between the asymptotes, through these points. **B1**

correct shape

(b) On $y = \tan x$: $\tan x = \frac{1}{\sqrt{3}}$ at $x = 30°$ and $30° + 180° = 210°$ **M1**

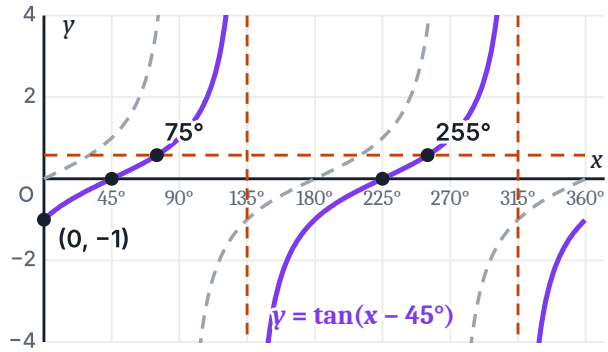
tan: period 180°

The graph is moved 45° to the right, so add 45° to each. **M1**

move the solutions right 45°

$x = 75°$, $x = 255°$ (both in the interval) **A1**

check the interval



asymptotes x = 135°, 315°; x = 75°, 255°