The Addition Formulae

Specification: AQA 7357: E6, E8 · Edexcel 9MA0: 5.6, 5.8

INDEPENDENT **You do**

A · Expanding No calculator. Give exact values.

1 Expand and simplify:
(a) $\sin(x + 30°)$
(b) $\cos(x + 30°)$
(c) $\sin(x - 45°)$
(d) $\cos(x - 60°)$

(a) $\sin x \cos 30° + \cos x \sin 30° = \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x$ — addition formula; exact values

(b) $\cos x \cos 30° - \sin x \sin 30° = \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$ — cos: sign flips

(c) $\sin x \cos 45° - \cos x \sin 45° = \frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x$ — sin(A − B)

(d) $\cos x \cos 60° + \sin x \sin 60° = \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x$ — cos(A − B): sign flips

2 Expand and simplify (angles in radians):
(a) $\sin(\theta + \frac{\pi}{3})$
(b) $\cos(\theta - \frac{\pi}{6})$
(c) $\sin(\theta + \frac{\pi}{2})$
(d) $\cos(\theta + \pi)$

(a) $\sin\theta\cos\frac{\pi}{3} + \cos\theta\sin\frac{\pi}{3} = \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta$ — sin(A + B); exact values

(b) $\cos\theta\cos\frac{\pi}{6} + \sin\theta\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta$ — cos(A − B): sign flips

(c) $\sin\theta\cos\frac{\pi}{2} + \cos\theta\sin\frac{\pi}{2} = \sin\theta \times 0 + \cos\theta \times 1 = \cos\theta$ — cos π/2 = 0, sin π/2 = 1

(d)
$\cos\theta\cos\pi - \sin\theta\sin\pi = \cos\theta \times (-1) - \sin\theta \times 0 = -\cos\theta$ — cos π = −1, sin π = 0

3 Write in terms of $\tan x$:
(a) $\tan(x + 45°)$
(b) $\tan(x - 60°)$

(a) $\frac{\tan x + \tan 45°}{1 - \tan x \tan 45°} = \frac{\tan x + 1}{1 - \tan x}$ — tan 45° = 1

(b) $\frac{\tan x - \tan 60°}{1 + \tan x \tan 60°} = \frac{\tan x - \sqrt{3}}{1 + \sqrt{3}\tan x}$ — tan 60° = √3

B · Backwards Write as a single trig ratio. Then find the exact value. No calculator.

4 (a) $\sin 40° \cos 20° + \cos 40° \sin 20°$
(b) $\cos 75° \cos 15° - \sin 75° \sin 15°$
(c) $\sin 100° \cos 55° - \cos 100° \sin 55°$
(d) $\cos 80° \cos 20° + \sin 80° \sin 20°$

(a) $\sin(40° + 20°) = \sin 60° = \frac{\sqrt{3}}{2}$ — sin(A + B) backwards

(b) $\cos(75° + 15°) = \cos 90° = 0$ — cos(A + B) backwards

(c) $\sin(100° - 55°) = \sin 45° = \frac{\sqrt{2}}{2}$ — sin(A − B) backwards

(d) $\cos(80° - 20°) = \cos 60° = \frac{1}{2}$ — cos(A − B) backwards

5 (a) $\frac{\tan 25° + \tan 20°}{1 - \tan 25° \tan 20°}$
(b) $\frac{\tan 100° - \tan 40°}{1 + \tan 100° \tan 40°}$

(a) $\tan(25° + 20°) = \tan 45° = 1$ — tan(A + B) backwards
(b) $\tan(100° - 40°) = \tan 60° = \sqrt{3}$ — tan(A − B) backwards

6 Write as a single trig ratio:
(a) $\sin 3\theta \cos\theta + \cos 3\theta \sin\theta$
(b) $\cos 5x \cos 2x + \sin 5x \sin 2x$
(c) $\sin(A + B)\cos B - \cos(A + B)\sin B$

(a) $\sin(3\theta + \theta) = \sin 4\theta$ — sin(A + B) backwards
(b) $\cos(5x - 2x) = \cos 3x$ — cos(A − B) backwards
(c) $\sin((A + B) - B) = \sin A$ — treat A + B as one angle

C · Reasoning No calculator.

7 Use $A = 60°$ and $B = 30°$ to show that $\cos(A - B)$ is not equal to $\cos A - \cos B$.

$\cos(60° - 30°) = \cos 30° = \frac{\sqrt{3}}{2} \approx 0.866$ — exact value

$\cos 60° - \cos 30° = \frac{1}{2} - \frac{\sqrt{3}}{2} \approx -0.366$ — exact values

Different, so the statement is false (a counter-example).

counter-example

8 Use the addition formulae to show that:
(a) $\sin(90° - \theta) \equiv \cos\theta$
(b) $\cos(180° + \theta) \equiv -\cos\theta$
(c) $\tan(\theta + 180°) \equiv \tan\theta$

(a) $\sin 90° \cos\theta - \cos 90° \sin\theta = 1 \times \cos\theta - 0 = \cos\theta$ — sin(A − B); exact values

(b) $\cos 180° \cos\theta - \sin 180° \sin\theta = -\cos\theta$ — cos(A + B); exact values

(c) $\frac{\tan\theta + \tan 180°}{1 - \tan\theta \tan 180°} = \frac{\tan\theta + 0}{1 - 0} = \tan\theta$ — tan 180° = 0

9 Put $A = B$ in the formula for $\cos(A - B)$. What identity do you get?

$\cos(A - A) = \cos A \cos A + \sin A \sin A$ — replace B with A

$\cos 0 = 1$ — cos 0 = 1

$1 \equiv \cos^2 A + \sin^2 A$ — Year 12 identity

D · Show that

10 Show that $\sin(x + 60°) + \sin(x - 60°) \equiv \sin x$.

$(\sin x \cos 60° + \cos x \sin 60°) + (\sin x \cos 60° - \cos x \sin 60°)$ — expand both

$= 2 \sin x \cos 60°$ — cos x sin 60° cancels

$= 2 \sin x \times \frac{1}{2} = \sin x$ = RHS — cos 60° = ½

11 Show that $\cos(x + 30°) - \cos(x - 30°) \equiv -\sin x$.

$(\cos x \cos 30° - \sin x \sin 30°) - (\cos x \cos 30° + \sin x \sin 30°)$ — expand both

$= -2 \sin x \sin 30° = -\sin x$ — sin 30° = ½

12 Show that $\frac{\sin(A + B)}{\cos A \cos B} \equiv \tan A + \tan B$.

$= \frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}$ — split the fraction

$= \frac{\sin A}{\cos A} + \frac{\sin B}{\cos B} = \tan A + \tan B$ — sin/cos = tan

13 Show that $\cos(x + \frac{\pi}{4}) + \sin(x + \frac{\pi}{4}) \equiv \sqrt{2}\cos x$.

$\cos(x + \frac{\pi}{4}) = \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x$ — addition formula

$\sin(x + \frac{\pi}{4}) = \frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\cos x$ — addition formula

Sum $= 2 \times \frac{\sqrt{2}}{2}\cos x = \sqrt{2}\cos x$ — add: sin terms cancel

E · Exam-style question AQA-style. Use the mark scheme to self-assess.

14 (a) Show that $\sin(\theta + 30°) + \cos(\theta + 60°) \equiv \cos\theta$. [3 marks]
(b) Hence solve $\sin(\theta + 30°) + \cos(\theta + 60°) = 0.4$ for $0° \le \theta < 360°$. Give your answers to 1 decimal place. [3 marks]

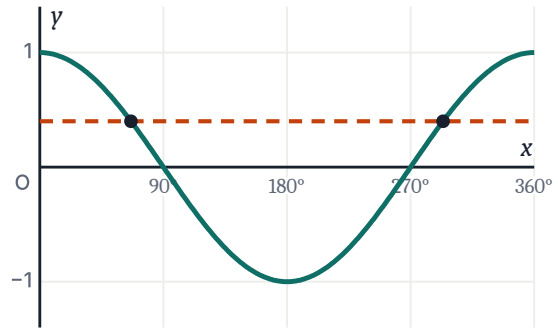
(a) $\sin\theta \cos 30° + \cos\theta \sin 30° + \cos\theta \cos 60° - \sin\theta \sin 60°$ M1 — expand both

$= \frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta + \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$ M1 — exact values

$= \cos\theta$ A1 — collect like terms

(b) $\cos\theta = 0.4 \Rightarrow \theta = \cos^{-1} 0.4 = 66.4°$ M1 A1 — calculator

$\theta = 360° - 66.4° = 293.6°$ A1 — cos: 360° − x



check: 2 crossings