Double-Angle Formulae

Specification: AQA 7357: E6, E5 · Edexcel 9MA0: 5.6, 5.5

INDEPENDENT ## You do

A · Single trig ratios No calculator.

1 Write each as a single trig ratio:

(a) $2\sin 40^\circ \cos 40^\circ$

(b) $\cos^2 70^\circ - \sin^2 70^\circ$

(c) $1 - 2\sin^2 35^\circ$

(d) $\dfrac{2\tan 10^\circ}{1-\tan^2 10^\circ}$

(e) $4\sin 3\theta \cos 3\theta$

(f) $\cos^2\dfrac{\theta}{2} - \sin^2\dfrac{\theta}{2}$

(a) $\sin(2\times 40^\circ) = \sin 80^\circ$ — 2 sin A cos A = sin 2A

(b) $\cos(2\times 70^\circ) = \cos 140^\circ$ — cos²A − sin²A = cos 2A

(c) $\cos(2\times 35^\circ) = \cos 70^\circ$ — 1 − 2 sin²A = cos 2A

(d) $\tan(2\times 10^\circ) = \tan 20^\circ$ — tan 2A backwards

(e) $2(2\sin 3\theta\cos 3\theta) = 2\sin(2\times 3\theta) = 2\sin 6\theta$ — take out 2

(f) $\cos(2\times\dfrac{\theta}{2}) = \cos\theta$ — A = θ/2

2 Find the exact value of:

(a) $2\sin 15^\circ \cos 15^\circ$

(b) $1 - 2\sin^2\dfrac{\pi}{12}$

(c) $\cos^2 22.5^\circ - \sin^2 22.5^\circ$

(d) $\dfrac{2\tan\dfrac{\pi}{12}}{1-\tan^2\dfrac{\pi}{12}}$

(a) $\sin(2\times 15^\circ) = \sin 30^\circ = \dfrac{1}{2}$ — sin 2A backwards

(b) $\cos(2\times\dfrac{\pi}{12}) = \cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$ — cos 2A backwards

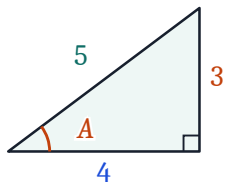
(c) $\cos(2\times 22.5^\circ) = \cos 45^\circ = \dfrac{\sqrt{2}}{2}$ — cos 2A backwards

(d) $\tan(2\times\dfrac{\pi}{12}) = \tan\dfrac{\pi}{6} = \dfrac{1}{\sqrt{3}}$ — tan 2A backwards

B · Given one ratio Exact answers.

3 $\sin A = \dfrac{3}{5}$, A acute. Find $\sin 2A$, $\cos 2A$ and $\tan 2A$.

$\cos^2 A = 1 - \dfrac{9}{25} = \dfrac{16}{25} \Rightarrow \cos A = \dfrac{4}{5}$ — acute: cos +



triangle 3, 4, 5

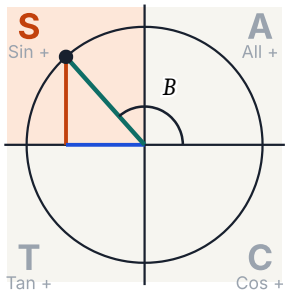
$\sin 2A = 2\times\dfrac{3}{5}\times\dfrac{4}{5} = \dfrac{24}{25}$ — sin 2A = 2 sin A cos A

$\cos 2A = 1 - 2\times\dfrac{9}{25} = \dfrac{7}{25}$ — cos 2A = 1 − 2 sin²A

$\tan 2A = \dfrac{\sin 2A}{\cos 2A} = \dfrac{\frac{24}{25}}{\frac{7}{25}} = \dfrac{24}{7}$ — tan = sin ÷ cos

4 $\cos B = -\dfrac{2}{3}$, B obtuse. Find $\sin 2B$ and $\cos 2B$.

$\sin^2 B = 1 - \dfrac{4}{9} = \dfrac{5}{9} \Rightarrow \sin B = \dfrac{\sqrt{5}}{3}$ — 2nd quadrant: sin +



CAST: where the angle lies

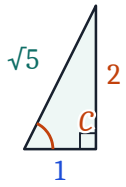
$\sin 2B = 2\times\dfrac{\sqrt{5}}{3}\times(-\dfrac{2}{3}) = -\dfrac{4\sqrt{5}}{9}$ — sin 2B = 2 sin B cos B

$\cos 2B = 2\times\dfrac{4}{9} - 1 = -\dfrac{1}{9}$ — cos 2B = 2 cos²B − 1

5 $\tan C = 2$, C acute. Find $\tan 2C$ and $\cos 2C$.

$\tan 2C = \dfrac{2\tan C}{1-\tan^2 C} = \dfrac{2\times 2}{1-2^2} = -\dfrac{4}{3}$ — tan 2C formula

Triangle 1, 2, $\sqrt{5}$: $\cos C = \dfrac{1}{\sqrt{5}}$ — right-angled triangle



triangle 1, 2, √5

$\cos 2C = 2\times\dfrac{1}{5} - 1 = -\dfrac{3}{5}$ — cos 2C = 2 cos²C − 1

6 Given that $\cos 2x = \dfrac{7}{25}$, find the possible values of $\sin x$.

$1 - 2\sin^2 x = \dfrac{7}{25}$ — only sin x wanted

$\sin^2 x = \dfrac{9}{25}$ — rearrange

$\sin x = \pm\dfrac{3}{5}$ — square root: ±

C · Reasoning

7 Show that $(\sin x + \cos x)^2 \equiv 1 + \sin 2x$.

$= \sin^2 x + 2\sin x\cos x + \cos^2 x$ — expand the bracket

$= 1 + \sin 2x$ — sin² + cos² = 1; double angle

8 Given that $\cos 2A = \dfrac{1}{9}$ and A is acute, find the exact value of $\cos A$.

$2\cos^2 A - 1 = \dfrac{1}{9}$ — cos 2A = 2 cos²A − 1

$\cos^2 A = \dfrac{5}{9}$ — rearrange

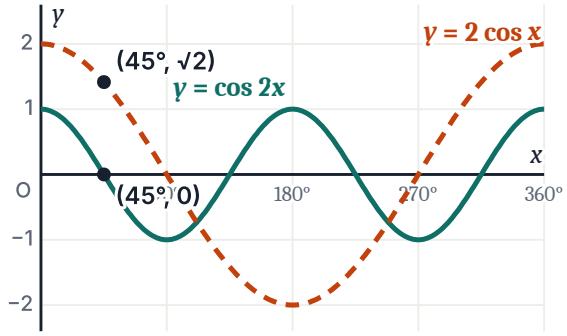
$\cos A = \dfrac{\sqrt{5}}{3}$ — acute: positive

9 Explain why the graph of $y = \cos 2x$ crosses the x-axis at $x = 45°$, but $y = 2\cos x$ does not.

$\cos(2\times 45°) = \cos 90° = 0$ — substitute x = 45°

$2\cos 45° = \sqrt{2} \neq 0$ — substitute x = 45°

$\cos 2x$ doubles the angle; $2\cos x$ doubles the height. — compare



compare the graphs

D · Algebra

10 Eliminate θ: $x = 3\cos\theta$, $y = 2\cos 2\theta$.

$\cos\theta = \dfrac{x}{3}$ — make cos θ the subject

$y = 2(2\cos^2\theta - 1) = 2(2\times\dfrac{x^2}{9} - 1)$ — double angle: cos form

$y = \dfrac{4x^2}{9} - 2$ — simplify

11 Eliminate θ: $x = \tan\theta$, $y = \tan 2\theta$.

$y = \dfrac{2\tan\theta}{1-\tan^2\theta}$ — tan 2θ formula

$y = \dfrac{2x}{1-x^2}$ — substitute

12 Given that $p = 2\sin\theta$ and $q = 3\cos 2\theta$, express q in terms of p.

$\sin\theta = \frac{p}{2}$ — make sin θ the subject

$q = 3(1 - 2\sin^2\theta) = 3(1 - \frac{p^2}{2})$ — double angle: sin form

$q = 3 - \frac{3p^2}{2}$ — simplify

13 Write $\cos^4 x - \sin^4 x$ as a single trig ratio.

$= (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$ — Do Now Q4

$= \cos 2x \times 1 = \cos 2x$ — $\sin^2 + \cos^2 = 1$

E · Exam-style question AQA-style. Use the mark scheme to self-assess.

14 θ is obtuse and $\cos\theta = -\frac{1}{4}$.

(a) Find the exact value of $\sin 2\theta$. [3 marks]

(b) Show that $\tan 2\theta = \frac{\sqrt{15}}{7}$. [3 marks]

(c) Find the exact value of $\cos 4\theta$. [2 marks]

(a) $\sin\theta = \frac{\sqrt{15}}{4}$ B1 — obtuse: sin +

$\sin 2\theta = 2 \times \frac{\sqrt{15}}{4} \times (-\frac{1}{4})$ M1 — sin 2θ = 2 sin θ cos θ

$= -\frac{\sqrt{15}}{8}$ A1 — simplify

(b) $\cos 2\theta = 2 \times \frac{1}{16} - 1 = -\frac{7}{8}$ M1 A1 — cos 2θ = 2 cos²θ − 1

$\tan 2\theta = \frac{-\frac{\sqrt{15}}{8}}{-\frac{7}{8}} = \frac{\sqrt{15}}{7}$ A1 — tan = sin ÷ cos

(c) $\cos 4\theta = 2\cos^2 2\theta - 1 = 2 \times \frac{49}{64} - 1 = \frac{17}{32}$ M1 A1 — double angle again