Solving Trigonometric Equations

Specification: AQA 7357: E6, E7 · Edexcel 9MA0: 5.6, 5.7

INDEPENDENT

You do

A · Multiple angles Exact answers where possible.

1 Solve for $0° \le x < 360°$:

(a) $\sin 2x = \frac{\sqrt{3}}{2}$

(b) $\cos 3x = 0$

(c) $\tan 2x = 1$

(d) $\sin \frac{x}{2} = \frac{1}{2}$

(a) $0° \le 2x < 720°$: $2x = 60°, 120°, 420°, 480° \Rightarrow$ $x = 30°, 60°, 210°, 240°$

change the interval; + 360°; ÷ 2

(b) $0° \le 3x < 1080°$: $3x = 90°, 270°, 450°, 630°, 810°, 990° \Rightarrow$ $x = 30°, 90°, 150°, 210°, 270°, 330°$

change the interval; + 180°; ÷ 3

(c) $0° \le 2x < 720°$: $2x = 45°, 225°, 405°, 585° \Rightarrow$ $x = 22.5°, 112.5°, 202.5°, 292.5°$

tan: + 180°; ÷ 2

(d) $0° \le \frac{x}{2} < 180°$: $\frac{x}{2} = 30°, 150° \Rightarrow x = 60°, 300°$

half angle: halve the interval; × 2

2 Solve for $0 \le x < 2\pi$. Give exact answers.

(a) $\cos 2x = -\frac{1}{2}$

(b) $\sin 3x = 1$

(a) $0 \le 2x < 4\pi$: $2x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$

change the interval; + 2π

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ ÷ 2

(b) $0 \le 3x < 6\pi$: $3x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$ sin = 1: + 2π each time

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ ÷ 3

B · Double-angle equations Solve for 0° ≤ x < 360° unless stated. Give non-exact answers to 1 decimal place.

3 (a) $\cos 2x = \cos x$

(b) $\cos 2x + \sin x = 0$

(a) $2\cos^2 x - 1 = \cos x \Rightarrow 2\cos^2 x - \cos x - 1 = 0$

cos 2x = 2 cos²x − 1

$(2\cos x + 1)(\cos x - 1) = 0$ factorise

$\cos x = -\frac{1}{2}$: 120°, 240°; $\cos x = 1$: 0° all solutions in range

(b) $1 - 2\sin^2 x + \sin x = 0 \Rightarrow 2\sin^2 x - \sin x - 1 = 0$

cos 2x = 1 − 2 sin²x

$(2\sin x + 1)(\sin x - 1) = 0$ factorise

$\sin x = -\frac{1}{2}$: 210°, 330°; $\sin x = 1$: 90° all solutions in range

4 $3\cos 2x + \cos x + 2 = 0$

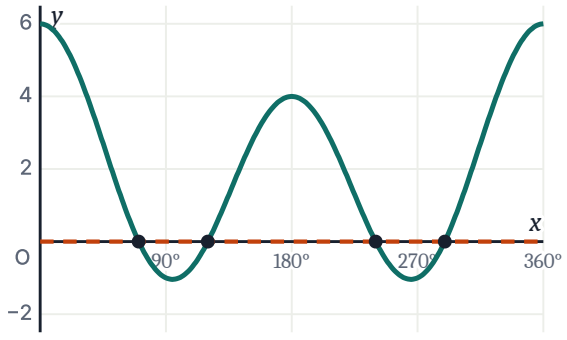
$6\cos^2 x - 3 + \cos x + 2 = 0 \Rightarrow 6\cos^2 x + \cos x - 1 = 0$

double angle: cos form

$(3\cos x - 1)(2\cos x + 1) = 0$ factorise

$\cos x = \frac{1}{3}$: 70.5°, 289.5° calculator; 360° − x

$\cos x = -\frac{1}{2}$: 120°, 240° CAST



check: 4 zeros

5 (a) $\sin 2x = \sin x$, $0° \le x \le 360°$
(b) $2 \sin 2x = 3 \cos x$

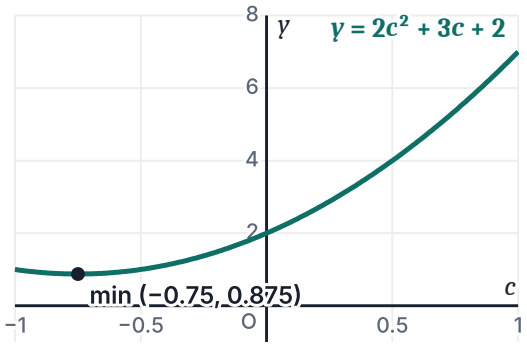
(a) $2 \sin x \cos x - \sin x = 0$ — sin 2x = 2 sin x cos x
$\sin x (2 \cos x - 1) = 0$ — factorise: never divide
$\sin x = 0$: $0°, 180°, 360°$; $\cos x = \frac{1}{2}$: $60°, 300°$
all solutions in range

(b) $4 \sin x \cos x - 3 \cos x = 0$ — sin 2x = 2 sin x cos x
$\cos x (4 \sin x - 3) = 0$ — factorise: never divide
$\cos x = 0$: $90°, 270°$; $\sin x = \frac{3}{4}$: $48.6°, 180° - 48.6° = 131.4°$
calculator; 180° − x

C · Reasoning

6 Show that $\cos 2x + 3 \cos x + 3 = 0$ has no real solutions.

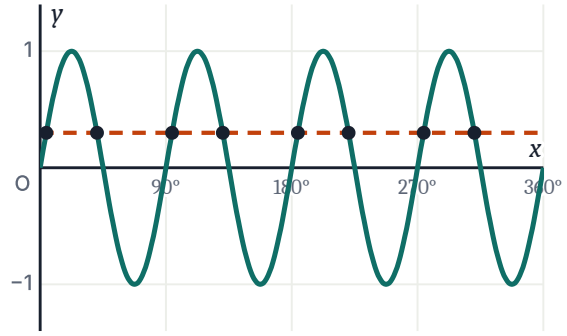
$2 \cos^2 x + 3 \cos x + 2 = 0$ — double angle; rearrange
Discriminant $3^2 - 4 \times 2 \times 2 = -7 < 0$ — $b^2 - 4ac$
No real values of $\cos x$, so no solutions. — no real roots



always above 0: no roots

7 How many solutions does $\sin 4x = 0.3$ have for $0° \le x < 360°$? Explain without solving.

$0° \le 4x < 1440°$: four complete waves. — change the interval
Each wave meets $y = 0.3$ twice: 8 solutions. — 2 per wave



8 crossings

8 A student solves $\tan 2x = 3 \tan x$ by dividing both sides by $\tan x$. Which solutions in $0° \le x < 360°$ are lost?

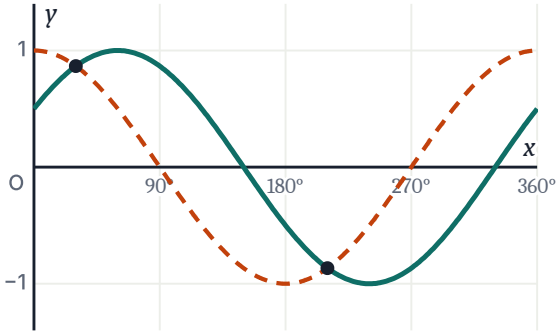
Dividing removes $\tan x = 0$. — tan x = 0 lost
$x = 0°$ and $x = 180°$ are lost. — Example 5

D · Addition formulae

Solve for 0° ≤ x < 360°. Give answers to 1 decimal place where needed.

9 $\sin(x + 30°) = \cos x$

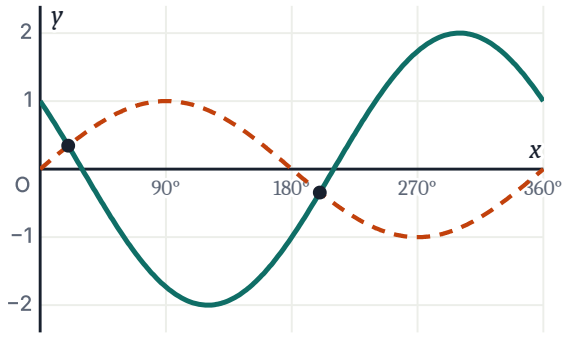
$\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x = \cos x$ — addition formula
$\frac{\sqrt{3}}{2} \sin x = \frac{1}{2} \cos x \Rightarrow \tan x = \frac{1}{\sqrt{3}}$ — divide by cos
$x = 30°, 210°$ — tan: + 180°



check: 2 crossings

10 $2 \cos(x + 60°) = \sin x$

$\cos x - \sqrt{3} \sin x = \sin x$ — addition formula
$\cos x = (1 + \sqrt{3}) \sin x \Rightarrow \tan x = \frac{1}{1 + \sqrt{3}} = 0.366...$
divide by sin and cos
$x = 20.1°, 200.1°$ — calculator; + 180°



check: 2 crossings

11 $\tan 2x = 4\tan x$

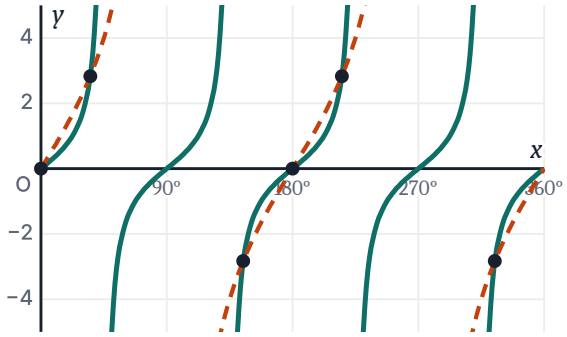
$\dfrac{2\tan x}{1-\tan^2 x} = 4\tan x$ — double angle for tan

$2\tan x = 4\tan x - 4\tan^3 x$ — × (1 − tan²x)

$2\tan x\,(2\tan^2 x - 1) = 0$ — factorise: never divide

$\tan x = 0$: $0°, 180°$ — tan x = 0

$\tan x = \pm\dfrac{1}{\sqrt{2}}$: $35.3°, 144.7°, 215.3°, 324.7°$ — square root: ±



check: 6 crossings

E · Exam-style question AQA-style. Use the mark scheme to self-assess.

12 (a) Show that the equation $3\cos 2x + 5\sin x = 4$ can be written as $6\sin^2 x - 5\sin x + 1 = 0$. [2 marks]
(b) Hence solve $3\cos 2x + 5\sin x = 4$ for $0° \le x < 360°$, giving your answers to 1 decimal place. [4 marks]

(a) $3(1 - 2\sin^2 x) + 5\sin x = 4$ M1 — double angle: sin form

$6\sin^2 x - 5\sin x + 1 = 0$ A1 — rearrange to = 0

(b) $(3\sin x - 1)(2\sin x - 1) = 0$ M1 — factorise

$\sin x = \dfrac{1}{3}, \sin x = \dfrac{1}{2}$ A1 — solve each factor

$x = 19.5°, 160.5°$ A1 — calculator; 180° − x

$x = 30°, 150°$ A1 — exact values