Writing a cos x + b sin x as R cos(x – α)

Specification: AQA 7357: E6, E3 · Edexcel 9MA0: 5.6, 5.3

INDEPENDENT

You do

A · Writing in R-form

R > 0. Give R exactly and α (0° < α < 90°) to 1 decimal place.

1 (a) $6\cos x + 8\sin x \equiv R\cos(x - \alpha)$
(b) $2\sin x + 3\cos x \equiv R\sin(x + \alpha)$
(c) $3\sin x - 2\cos x \equiv R\sin(x - \alpha)$
(d) $4\cos x - 7\sin x \equiv R\cos(x + \alpha)$

2 Give exact values of R and α, with $0 < \alpha < \frac{\pi}{2}$:
(a) $\sin x + \cos x \equiv R\sin(x + \alpha)$
(b) $\sqrt{3}\cos x + \sin x \equiv R\cos(x - \alpha)$
(c) $\cos x - \sin x \equiv R\cos(x + \alpha)$

B · Maximum and minimum

Use the R-form first.

3 (a) Write $24\sin x + 7\cos x$ in the form $R\sin(x + \alpha)$.
(b) State its maximum value and the value of x in $0° \le x < 360°$ where it occurs.

4 (a) Write $f(x) = 2\cos x + 2\sqrt{3}\sin x$ in the form $R\cos(x - \alpha)$.
(b) Find the maximum and minimum of $f(x)$ and where they occur in $0° \le x < 360°$.

5 Find the maximum and minimum values of:
(a) $g(x) = 10 + 3\cos x + 4\sin x$
(b) $h(x) = \dfrac{1}{7 + 3\cos x - 4\sin x}$

C · Reasoning

6 Explain why the equation $5\sin x + 12\cos x = 14$ has no solutions.

7 Without working out α, which has the larger maximum: $2\sin x + 3\cos x$ or $3\sin x + 2\cos x$?

8 $3\cos 2x + 4\sin 2x$: state its maximum value and how many times it reaches it for $0° \le x < 360°$.

D · Sketching

9 (a) Write $\sin x + \cos x$ in the form $R\sin(x + \alpha)$.
(b) Sketch $y = \sin x + \cos x$ for $0° \le x \le 360°$. Label the turning points and the points where it meets the axes.

10 Sketch $y = 3\cos x + 4\sin x$ for $0° \le x \le 360°$. Label the turning points and the points where it meets the axes.

E · Exam-style question

AQA-style. Use the mark scheme to self-assess.

11 (a) Express $6\sin x + 2.5\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give α to 1 decimal place. [3 marks]
(b) State the maximum value of $6\sin x + 2.5\cos x$ and the smallest positive value of x at which it occurs. [2 marks]
(c) Find the minimum value of $\dfrac{1}{8 - 6\sin x - 2.5\cos x}$. [2 marks]