Solving Equations Using R-Form

Specification: AQA 7357: E6, E7 · Edexcel 9MA0: 5.6, 5.7

INDEPENDENT **You do**

A · Solving Solve for $0° \le x < 360°$ unless stated. Give answers to 1 decimal place.

1 Use $3\cos x + 4\sin x \equiv 5\cos(x - 53.13°)$ to solve:
(a) $3\cos x + 4\sin x = 1$
(b) $3\cos x + 4\sin x = -2.5$
(c) $3\cos x + 4\sin x = 5$

2 (a) $2\sin x + 3\cos x = 1$
(b) $5\cos x - 2\sin x = 4$

3 Solve for $0 \le x < 2\pi$. Give exact answers.
(a) $\sin x + \cos x = \frac{1}{\sqrt{2}}$
(b) $\sqrt{3}\cos x + \sin x = 1$

B · Maximum and minimum

4 $f(x) = \frac{6}{4 + \sin x + \sqrt{3}\cos x}$, $0 \le x < 2\pi$. Find the maximum and minimum values of $f(x)$ and where they occur.

5 Find the maximum and minimum values of:
(a) $(3\cos x + 4\sin x)^2$
(b) $5 + 2(5\sin x - 12\cos x)$

C · Reasoning

6 For which values of k does $3\cos x + 4\sin x = k$ have solutions?

7 Explain why squaring $\cos x + \sin x = 1$ gives four answers in $0° \le x < 360°$, but only two of them work.

8 How many solutions does $5\sin 2x + 12\cos 2x = 6$ have for $0° \le x < 360°$? Explain without solving.

D · Longer problems

9 Solve $2\sin 2x + \cos 2x = 1$ for $0° \le x < 180°$:
(a) using the double-angle formulae
(b) using the R-form. Check your answers agree.

10 $f(\theta) = 4\cos\theta - 3\sin\theta$.
(a) Write $f(\theta)$ in the form $R\cos(\theta + \alpha)$, where $0° < \alpha < 90°$.
(b) Solve $f(\theta) = 1$ for $0° \le \theta < 360°$.

E · Exam-style question AQA-style. Use the mark scheme to self-assess.

11 (a) Express $2\cos x - 5\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give α to 2 decimal places. [3 marks]
(b) Hence solve $2\cos x - 5\sin x = 3$ for $0° \le x < 360°$, giving your answers to 1 decimal place. [4 marks]
(c) Write down the maximum value of $\frac{1}{10 + 2\cos x - 5\sin x}$. [2 marks]