Proving Trigonometric Identities

Specification: AQA 7357: E8, E6, E5 · Edexcel 9MA0: 5.8, 5.6, 5.5

INDEPENDENT

You do

A · One or two steps Prove each identity.

1 (a) $\dfrac{\sin 2x}{\sin x} \equiv 2\cos x$

(b) $\dfrac{\cos 2x + 1}{2\cos x} \equiv \cos x$

2 (a) $\dfrac{2\sin x \cos x}{\cos^2 x - \sin^2 x} \equiv \tan 2x$

(b) $\dfrac{1 - \cos 2x}{1 + \cos 2x} \equiv \tan^2 x$

3 (a) $\sin 2x \tan x + \cos 2x \equiv 1$

(b) $2\operatorname{cosec} 2x - \cot x \equiv \tan x$

B · Several steps Prove each identity.

4 $\cot x - \cot 2x \equiv \operatorname{cosec} 2x$

5 $\dfrac{\sec^2 x}{2 - \sec^2 x} \equiv \sec 2x$

6 $\dfrac{\cos x + \sin x}{\cos x - \sin x} \equiv \dfrac{1 + \sin 2x}{\cos 2x}$

C · Prove or disprove

7 Prove or disprove: $(\sin x + \cos x)^2 \equiv 1$.

8 Prove or disprove: $\cos 2x \equiv 2\cos x - 1$.

9 Prove or disprove: $\tan 2x \equiv 2\tan x$.

D · Addition and triple angles

10 Use $\cos 3x \equiv 4\cos^3 x - 3\cos x$ to prove that $\dfrac{\cos 3x}{\cos x} \equiv 2\cos 2x - 1$.

11 Prove that $\dfrac{\sin(A+B) + \sin(A-B)}{\cos(A+B) + \cos(A-B)} \equiv \tan A$.

12 Prove that $4\sin(x + 30°)\sin(x - 30°) \equiv 1 - 2\cos 2x$.

E · Exam-style question AQA-style. Use the mark scheme to self-assess.

13 (a) Show that $\dfrac{\cos 2\theta}{\cos\theta + \sin\theta} \equiv \cos\theta - \sin\theta$. [2 marks]

(b) Hence solve $\dfrac{\cos 2\theta}{\cos\theta + \sin\theta} = 0.5$ for $0° \le \theta < 360°$. Give your answers to 1 decimal place. [5 marks]