# Modelling with Trigonometric Functions

Specification: AQA 7357: E9, E6, E7 · Edexcel 9MA0: 5.9, 5.6, 5.7

**INDEPENDENT** **You do**

## A · Reading a model

**1** The height of a seat on a wheel is $h = 21 - 19\cos(9t)°$ m, $t$ minutes after the start.
(a) Find the maximum and minimum heights.
(b) Find the time for one revolution.
(c) Find the height after 5 minutes.

(a) Max $21 + 19 = 40$ m (when $\cos(9t)° = -1$); min $21 - 19 = 2$ m (when $\cos(9t)° = 1$)

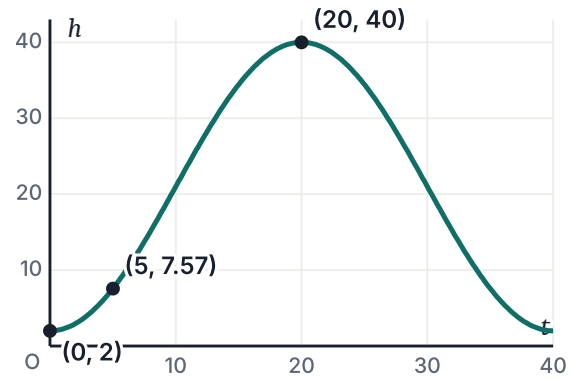
p ± q

(b) $\frac{360}{9} = 40$ minutes

period: 360 ÷ k

(c) $h = 21 - 19\cos(9 \times 5)° = 21 - 19\cos 45° = 7.57$ m

substitute t = 5




max, min; t = 5

**2** The depth of water is $D = 5 + 3\sin(\frac{\pi t}{6})$ m, $t$ hours after midnight (radians).
(a) Find the greatest and least depths.
(b) Find the period.
(c) Find the depth at 07:30.

(a) Greatest $5 + 3 = 8$ m; least $5 - 3 = 2$ m

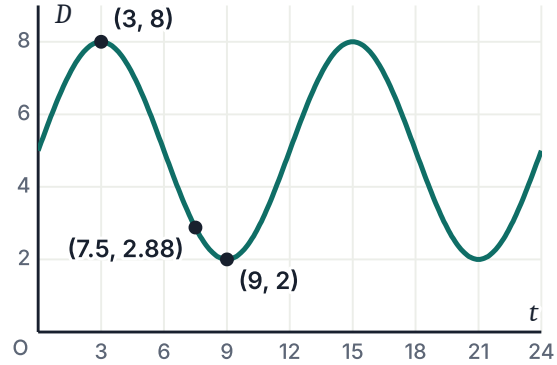
p ± q

(b) $\frac{2\pi}{\frac{\pi}{6}} = 12$ hours

period: 2π ÷ k

(c) $t = 7.5$: $5 + 3\sin(1.25\pi) = 2.88$ m

radians: substitute t = 7.5




max, min; t = 7.5

## B · Solving Give times in context.

**3** For $h = 21 - 19\cos(9t)°$, find the two times in the first revolution when the seat is 30 m high.

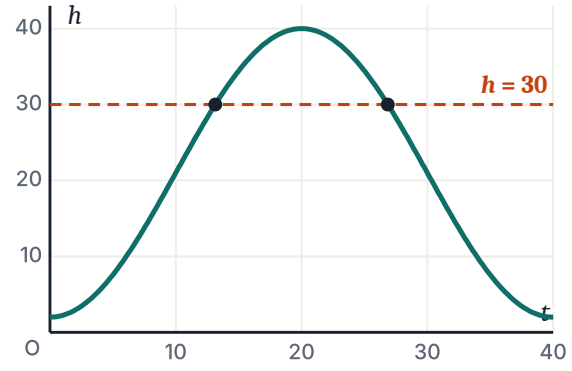
$21 - 19\cos(9t)° = 30 \Rightarrow \cos(9t)° = -\frac{9}{19}$

set the model equal to 30

First revolution $0 \le 9t < 360$:
$9t = 118.27°, 360° - 118.27° = 241.73°$

calculator; 360° − x ÷ 9; in context

$t = 13.1, 26.9$ minutes




2 crossings of h = 30

**4** For $D = 5 + 3\sin(\frac{\pi t}{6})$, find all the times in a 24-hour day when the depth is 6.5 m.

$5 + 3\sin(\frac{\pi t}{6}) = 6.5 \Rightarrow \sin(\frac{\pi t}{6}) = 0.5,\ 0 \le \frac{\pi t}{6} < 4\pi$

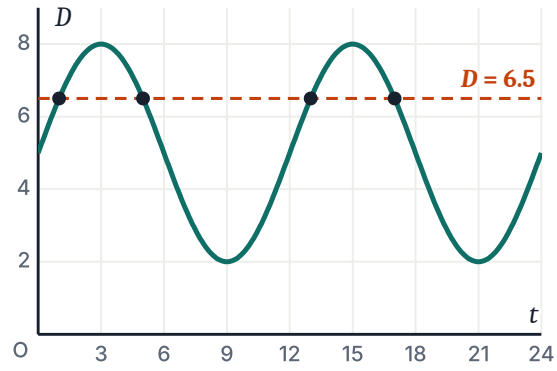
set equal; full interval

$\frac{\pi t}{6} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

all solutions in range

$t = 1, 5, 13, 17$: 01:00, 05:00, 13:00, 17:00

convert to times




4 crossings of D = 6.5

**5** A model for the temperature on a summer day is $T = 15 + 7\sin(15(t-9))°$ °C, $t$ hours after midnight. For how long is the temperature above 18 °C?

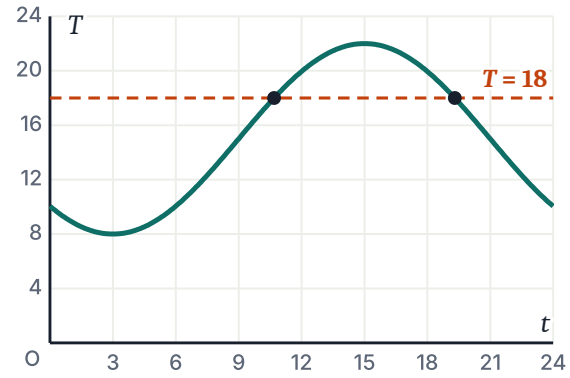
$15 + 7\sin(15(t-9))° = 18 \Rightarrow \sin(15(t-9))° = \frac{3}{7}$

set the model equal

$15(t-9) = 25.38°, 180° - 25.38° = 154.62°$ calculator; 180° − x

$t - 9 = 1.692, 10.308 \Rightarrow t = 10.692, 19.308$ solve for t

$19.308 - 10.692 = 8.62$ hours ≈ 8 h 37 min difference of times




above 18 °C between the dots

## C · Building models

**6** A wheel has diameter 120 m and its centre is 70 m above the ground. It turns once every 30 minutes, starting with a seat at the bottom. Find a model for the height $H$ m of the seat after $t$ minutes.

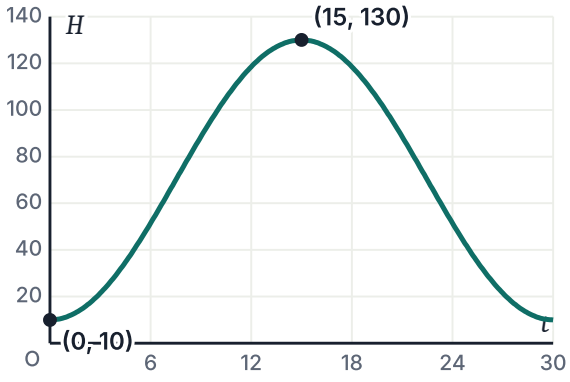
Centre height $p = 70$; radius $q = \frac{120}{2} = 60$; $k = \frac{360}{30} = 12$

centre, radius, 360 ÷ period

Starts at the bottom, so use $H = p - q\cos(kt)°$

starts at bottom: − cos

$H = 70 - 60\cos(12t)°$ write the model




check the model

**7** High tide is 9 m at 01:00 and low tide is 3 m at 07:00. The pattern repeats every 12 hours.
(a) Find a model $D = p + q\cos(k(t-c))°$.
(b) Give one limitation of your model.

(a) $p = \frac{9+3}{2} = 6, q = \frac{9-3}{2} = 3$ mean and amplitude

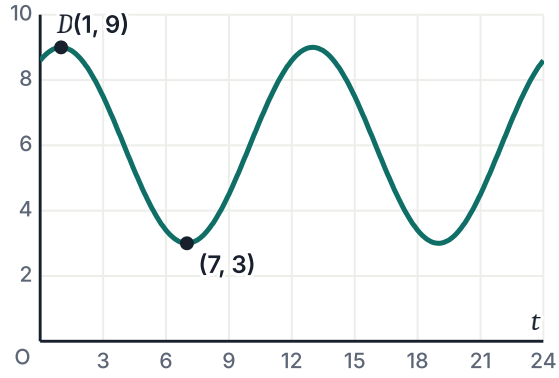
$k = \frac{360}{12} = 30$; maximum at $t = 1$, so $c = 1$

k from the period; max at t = c

$D = 6 + 3\cos(30(t-1))°$ write the model

(b) e.g. real tides repeat about every 12.4 h, so the model drifts; weather changes the depth.

limitation




check the model

## D · R-form models

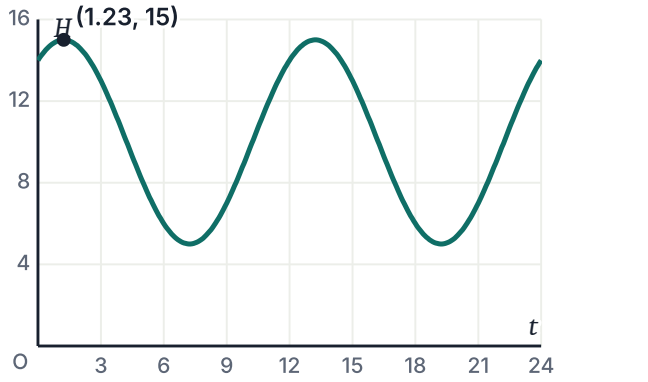
**8** A model for the depth of a river is $H = 10 + 3\sin(30t)° + 4\cos(30t)°$ m, $t$ hours after midnight.
(a) Write $3\sin(30t)° + 4\cos(30t)°$ in the form $R\sin(30t + \alpha)°$.
(b) Find the greatest depth and the first time it occurs.

(a) $R\cos\alpha = 3, R\sin\alpha = 4 \Rightarrow R = 5, \tan\alpha = \frac{4}{3} \Rightarrow \alpha = 53.13$
compare; square and add; divide

$3\sin(30t)° + 4\cos(30t)° = 5\sin(30t + 53.13)°$
write the R-form

(b) Greatest $10 + 5 = 15$ m when $30t + 53.13 = 90$
max: bracket = 90°

$t = \frac{36.87}{30} = 1.229$ hours = 1 h 14 min: 01:14
convert to a time



greatest depth

**9** The brightness of a star is modelled by $B = 50 - 12\cos(0.5t) + 5\sin(0.5t)$, where $t$ is in days (radians).
(a) Write $12\cos(0.5t) - 5\sin(0.5t)$ in the form $R\cos(0.5t + \alpha)$, with $\alpha$ in radians.
(b) Find the greatest brightness and the first time it occurs.

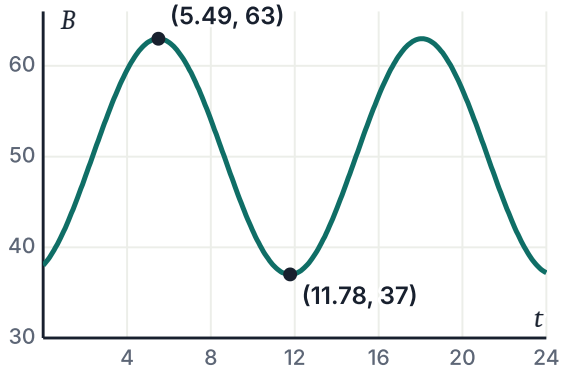
(a) $R\cos\alpha = 12, R\sin\alpha = 5 \Rightarrow R = \sqrt{144 + 25} = 13$
compare; square and add

$\tan\alpha = \frac{5}{12} \Rightarrow \alpha = 0.3948$ (radians)
divide: tan α (radians)

$B = 50 - 13\cos(0.5t + 0.3948)$
rewrite the model

(b) Greatest $50 + 13 = 63$ when $\cos(0.5t + 0.3948) = -1$
max: cos = −1

$0.5t + 0.3948 = \pi \Rightarrow t = 5.49$ days
solve for t



greatest brightness

## E · Exam-style question AQA-style. Use the mark scheme to self-assess.

**10** The height above the water of a point on a waterwheel is $h = 2.5 + 4\sin(24t)° - 3\cos(24t)°$ metres, where $t$ is the time in seconds.
(a) Express $4\sin(24t)° - 3\cos(24t)°$ in the form $R\sin(24t - \alpha)°$, where $0 < \alpha < 90$. [3 marks]
(b) Find the maximum height and the first time it occurs. [2 marks]
(c) For how long in each revolution is the point under the water? [4 marks]
(d) Give one limitation of the model. [1 mark]

**(a)** $R\cos\alpha = 4, R\sin\alpha = 3$ **M1**
compare coefficients

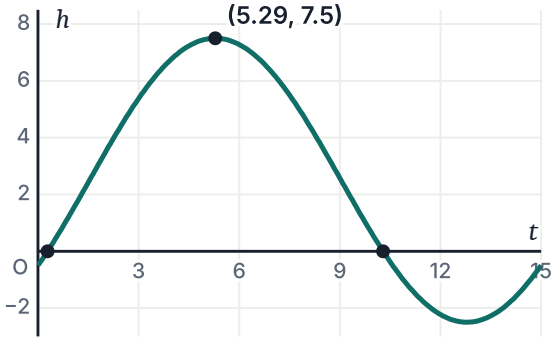
$R = \sqrt{16 + 9} = 5, \tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.87$ **A1 A1**
square and add; divide

**(b)** Max 7.5 m when $24t - 36.87 = 90$: $t = 5.29$ s **B1 B1**
max: bracket = 90°

**(c)** $h < 0 \Rightarrow \sin(24t - 36.87)° < -0.5$ **M1**
set h < 0

$24t - 36.87$ between 210° and 330° **M1 A1**
sin < −0.5: 210° to 330°

Under water for $\frac{120}{24} = 5$ seconds each revolution **A1**
÷ 24



under water: below the t-axis

**(d)** e.g. the water level changes, so the centre height is not fixed. **B1**
limitation