Modelling with Trigonometric Functions

Specification: AQA 7357: E9, E6, E7 · Edexcel 9MA0: 5.9, 5.6, 5.7

INDEPENDENT

You do

A · Reading a model

1 The height of a seat on a wheel is $h = 21 - 19\cos(9t)°$ m, t minutes after the start.
(a) Find the maximum and minimum heights.
(b) Find the time for one revolution.
(c) Find the height after 5 minutes.

2 The depth of water is $D = 5 + 3\sin(\frac{\pi t}{6})$ m, t hours after midnight (radians).
(a) Find the greatest and least depths.
(b) Find the period.
(c) Find the depth at 07:30.

B · Solving Give times in context.

3 For $h = 21 - 19\cos(9t)°$, find the two times in the first revolution when the seat is 30 m high.

4 For $D = 5 + 3\sin(\frac{\pi t}{6})$, find all the times in a 24-hour day when the depth is 6.5 m.

5 A model for the temperature on a summer day is $T = 15 + 7\sin(15(t - 9))°$ °C, t hours after midnight. For how long is the temperature above 18 °C?

C · Building models

6 A wheel has diameter 120 m and its centre is 70 m above the ground. It turns once every 30 minutes, starting with a seat at the bottom. Find a model for the height H m of the seat after t minutes.

7 High tide is 9 m at 01:00 and low tide is 3 m at 07:00. The pattern repeats every 12 hours.
(a) Find a model $D = p + q\cos(k(t - c))°$.
(b) Give one limitation of your model.

D · R-form models

8 A model for the depth of a river is $H = 10 + 3\sin(30t)° + 4\cos(30t)°$ m, t hours after midnight.
(a) Write $3\sin(30t)° + 4\cos(30t)°$ in the form $R\sin(30t + \alpha)°$.
(b) Find the greatest depth and the first time it occurs.

9 The brightness of a star is modelled by $B = 50 - 12\cos(0.5t) + 5\sin(0.5t)$, where t is in days (radians).
(a) Write $12\cos(0.5t) - 5\sin(0.5t)$ in the form $R\cos(0.5t + \alpha)$, with α in radians.
(b) Find the greatest brightness and the first time it occurs.

E · Exam-style question AQA-style. Use the mark scheme to self-assess.

10 The height above the water of a point on a waterwheel is $h = 2.5 + 4\sin(24t)° - 3\cos(24t)°$ metres, where t is the time in seconds.
(a) Express $4\sin(24t)° - 3\cos(24t)°$ in the form $R\sin(24t - \alpha)°$, where $0 < \alpha < 90$. [3 marks]
(b) Find the maximum height and the first time it occurs. [2 marks]
(c) For how long in each revolution is the point under the water? [4 marks]
(d) Give one limitation of the model. [1 mark]